2. Strong interaction – basic concepts

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Strong interaction between hadrons (quarks) is mediated by gluons as is shown in Fig.1.

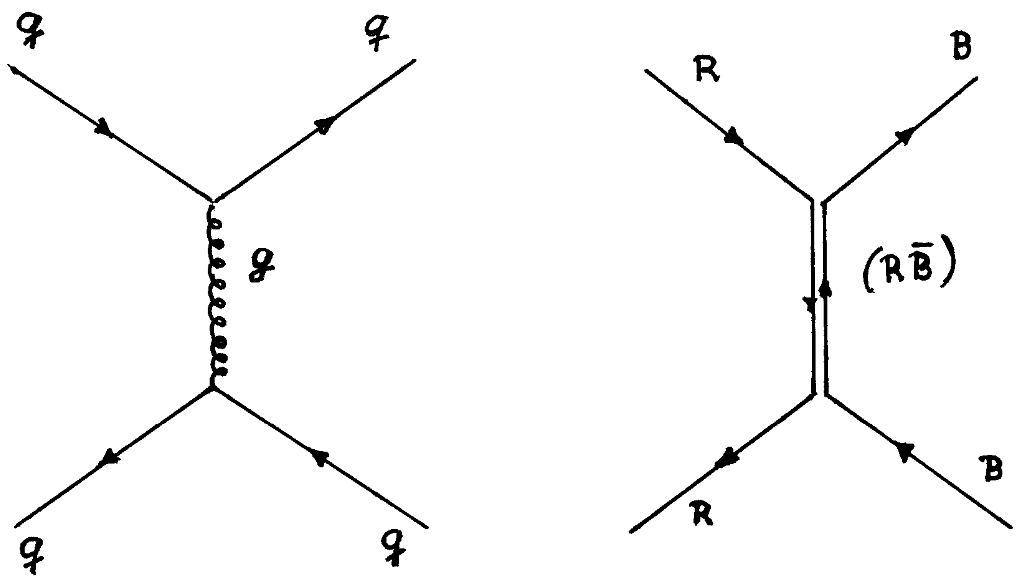


Fig. 1 : Scheme of strong interactions – gluon exchange; flow of colored charge at strong interaction.

The basic features of strong interactions are:

- There exit 3 type of strong interaction charge that is called color. The color charge states of quark are: $|R\rangle, |G\rangle, |B\rangle$ (red, green, blue).
- Quanta of strong interactions are gluons – there is 8 gluons: $\boldsymbol{g_i}$ ***i=1...8.***

Basic characteristics of gluons:

- Gluons are bosons (spin S=1), they are massless ($m_g=0$) and present colored bi-combinations
- From 9 possible bi-combinations one is singlet, i.e. it has no colored charge:

$$\frac{1}{\sqrt{3}}\left(R\overline{R}+G\overline{G}+B\overline{B}\right) \qquad (2.1)$$

- In contrast to photons the gluons can interact between them (as they have colored charge).

2.1 Comparison of coupling constant of strong and electromagnetic interactions.

Coupling constant characterizes strongness of interaction – e.g. the electro-magnetic interaction is characterized by coupling constant:

$$\alpha_{em} = \frac{e^2}{4\pi} \tag{2.2}$$

where e is the elementary electric charge.

An important information about the relation of strong (α_s) and electromagnetic (α_{em}) coupling constant is provided by decays (strong and electromagnetic) of unstable baryons. We will relay on the fact that decay half-width (Γ) is connected to coupling constant by the relation:

$$\Gamma \sim \alpha^2 \tag{2.3}$$

Let us consider the decay of resonance $\Sigma^0(1385)$:

$$K^- + p \rightarrow \Sigma^0(1385) \rightarrow \Lambda + \pi^0$$

This resonance, v this case, is produced in *Kp*-interactions and decays due to strong interaction. The life time of this resonance is:

$$\tau = \hbar/\Gamma \approx 10^{-23}\, s \tag{2.4}$$

Let us compare the above-mentioned decay with the electromagnetic one of $\Sigma^0(1192)$:

$$\Sigma^0(1192) \rightarrow \Lambda + \gamma \qquad \text{with life-time } \tau \approx 10^{-19}\, s.$$

The result of comparison is:

$$\frac{\alpha_s}{\alpha_{em}} = \left(\frac{10^{-19}}{10^{-23}}\right)^{1/2} \approx 100 \tag{2.5}$$

So, the strong interaction is really strong!

Remark. For comparison of coupling constants is important to have both compared processes running with a comparable transferred momentum, because (as we will see later) coupling constant depends on transferred momentum characterizing a given process. In our case this condition is fulfilled as the kinetic energy transferred to the output channel particles is in the above considered processes practically the same.

2.2 Global comparison of strong and electromagnetic interaction

***Electromagnetic interaction*.** Electric charge in vacuum (e.g. electron) makes a polarization of the vacuum in accordance with the scheme:

$$e^{-} \leftrightarrow \textit{virtual photon } (\gamma) \leftrightarrow e^{+}e^{-}$$

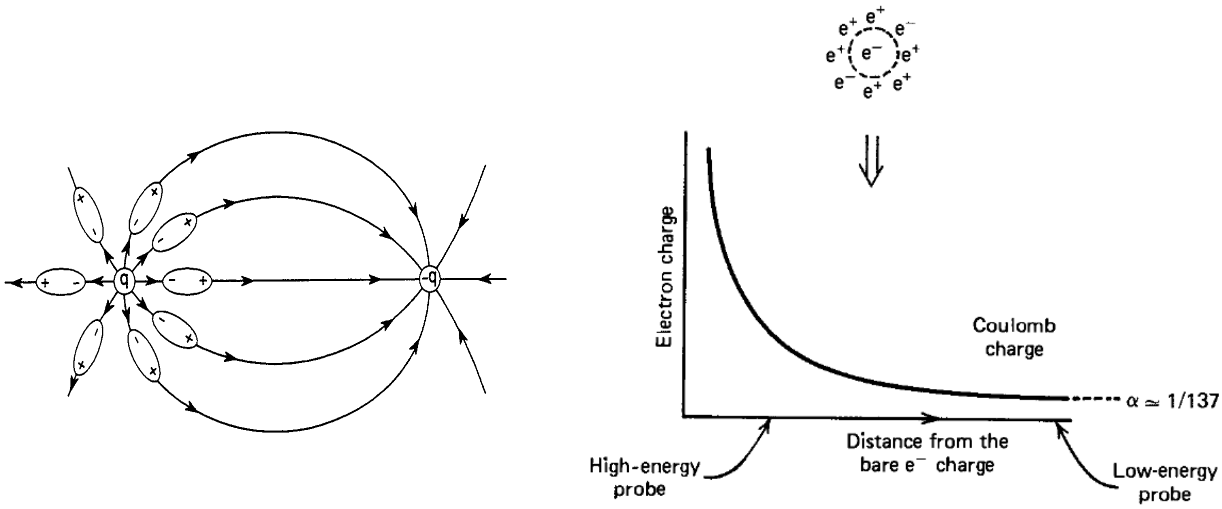


Fig. 2: Polarization of vacuum (= screening of charge) – diminution of coupling constant with increasing distance.

A consequence of the vacuum polarization:

Electron is surrounded by a cloud of virtual pairs of $e^{+}e^{-}$, which screens his charge. If we put into the field of electron charge a test unit charge then effective charge of electron obtained using Coulomb law, is a decreasing function of distance (see Fig. 2).

The elementary electric charge present in the relation for the electromagnetic coupling constant ***(*α_{em}*=1/137)*** is a charge measured in Thompson scattering. i.e. at large distances (at small transferred momenta).

***Strong interaction*.** Colored charge of quark polarizes the vacuum in accordance with the scheme (q≡quark, g≡gluon):

$$q \leftrightarrow virt.\,g \leftrightarrow \begin{cases} virt.\,q\bar{q} \\ \\ virt.\,gg \end{cases}$$

The difference from the electron case: virtual gluon can create also *gg*-pairs, what is a consequence of colored charge of gluon.

Consequence: In case of the strong interactions, unlike of the electromagnetic ones, occurs an anti-screening of colored charge.

Reason: Cloud of colored $q\bar{q}$ - pairs leads to an increase of α_s at low distances (like in the electromagnetic case). However cloud of virtual gluon leads to a decrease of α_s at $r \to 0$.

The overall effect in system with 8 gluons and not more than 16 flavors is that the coupling constant α_s is decreasing **($\alpha_s \to 0$)** at low distances (***$r \to 0$***). This effect is called ***asymptotic freedom.***

Confinement of quarks.

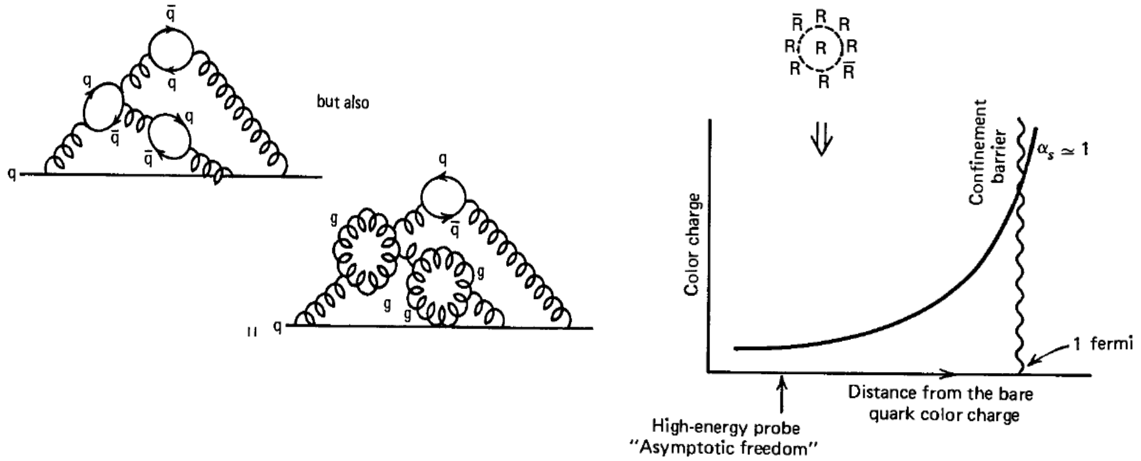


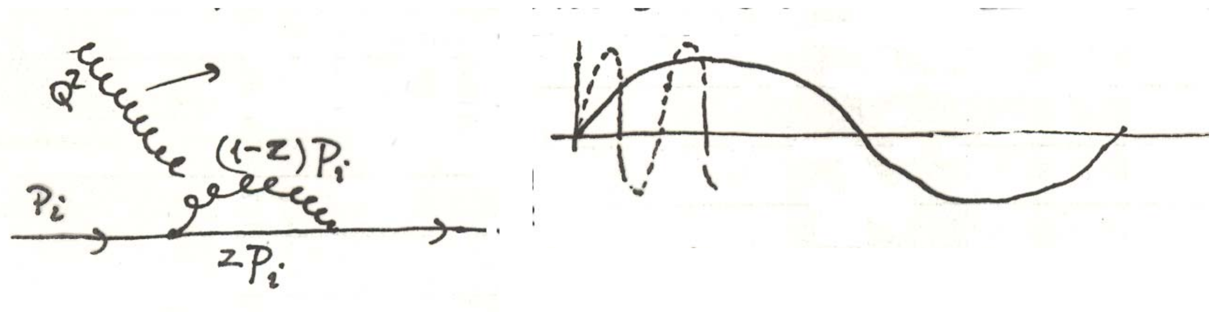
Fig. 3: Confinement of quarks: potential energy increases with increasing distance of quarks

With increasing distance of interacting quarks α_s increases at the same time the potential of interacting quarks it is possible to approximate by potential of harmonic oscillator ***$V(r) = \lambda r$*** (see Fig. 3). If distance between quarks achieve the value of *1 fm* then accumulated potential energy is sufficient for creation of a new pair $q\bar{q}$ and instead one original pair we have two pairs. This phenomenon is called the ***confinement of quarks***.

How to make small and big distances. Let us consider an interaction of a „point-like“ particle intermediated by an intermediate boson (see Fig. 1). A basic kinetic characteristic of such an interaction is transferred 4-momentum, i.e. 4-momentum virtual intermediate boson (***q***) or its 3-dimensional momentum (***Q***):

$$\boldsymbol{q}^2 = (\boldsymbol{p} - \boldsymbol{p}')^2 = (\boldsymbol{k} - \boldsymbol{k}')^2 \qquad \textit{resp.} \quad \boldsymbol{Q}^2 = -\boldsymbol{q}^2 \tag{2.6}$$

where p a k (p' a k') are 4-momenta of particles entering into (or outputting from) interaction.



However to a particle with the momentum ***Q*** ($= |\vec{Q}|$) can be associated a wave with wave length $\lambda = 2\pi/Q$. Therefore intermediated boson with high ***Q*** „scans“ small distances, while that with small ***Q*** „scans“ big distances.

2.3 The reasons for introducing colored charge.

Baryon $\boldsymbol{\Delta^{++}}$. Let us take into account baryon Δ^{++} (quark structure: *uuu*). It is a particle (resonance) with spin 3/2 representing a system of 3 identical fermions. The wave function of Δ^{++} must be therefore anti-symmetric. If we do not assume the existence of colored charge then for the wave function of Δ^{++} we get:

$$\boldsymbol{\Psi(1,2,3) = \chi(3/2, s_z) \cdot \varphi(\vec{r}_1, \vec{r}_2, \vec{r}_3)} \tag{2.7}$$

The spin wave function $\chi(3/2, s_z)$ is symmetric (rules of adding spins).

As the quarks *u u u* are in basic state with orbital momentum $L=0$, hence also $\boldsymbol{\varphi(\vec{r}_1 . \vec{r}_2, \vec{r}_3)}$ must be symmetric. ***Consequence:*** also the full wave function $\boldsymbol{\Psi}$ is symmetric and thereby baryon Δ^{++} does not obey the Fermi statistics!

Solution of the problem: Quarks have an additional degree of freedom – colored charge (color) and the part of the wave function corresponding to this degree of freedom is anti-symmetric.

The anti-symmetricity can be obtained in such a way that colored charge will acquire (at least) 3 values ($Q_1=R$, $Q_2=G$, $Q_3=B$). Only in this case for 3 quarks we can create the fully anti-symmetric colored wave function component:

$$\Psi_c = \frac{1}{\sqrt{6}} \varepsilon^{ijk} \cdot Q_i Q_j Q_k \tag{2.8}$$

where ε^{ijk} is unit anti-symmetric tensor.

Hadronic production in e^+e^-–beams. A direct test of the number of colors can be obtained from the ratio

$$R_{e^+e^-} \equiv \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} \tag{2.9}$$

Hadron production is realized through $e^+e^- \to \gamma^*, Z^* \to q\bar{q} \to hadrons$ (see Fig. 4) and as quarks with probability 1 hadronize, the sum over all quarks in final state gives the total cross

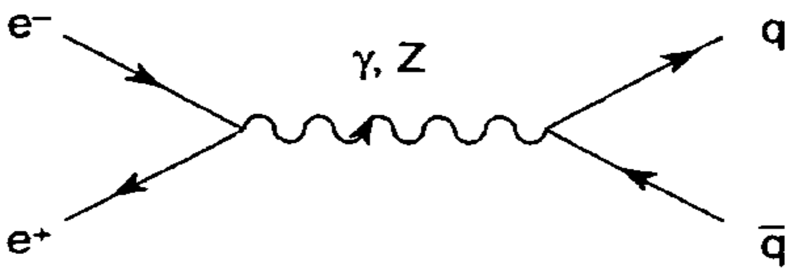


Fig. 4: Diagram for hadron production: $e^+ e^- \to$hadrons

inclusive section for hadron production. At energies << m_Z the γ-exchange is dominant and the ratio $R_{e^+e^-}$ reads:

$$R_{e^+e^-} \approx N_C \sum_{f=1}^{N_f} Q_f^2 = \begin{cases} \frac{2}{3} N_C = 2 & (N_f = 3: \ u,d,s) \\ \frac{10}{9} N_C = \frac{10}{3} & (N_f = 4: \ u,d,s,c) \\ \frac{11}{9} N_C = \frac{11}{3} & (N_f = 5: \ u,d,s,c,b) \end{cases} \tag{2.10}$$

The measured ratio is shown in Fig. 5. Though the simple formula 2.10 could not explain fully the complicated structure of the mentioned ratio it gives the correct mean values of the cross sections (except of the production threshold of individual quarks).

It should be noted that the number of hadron (quark) channel is $N_f \times N_C$, where N_f is the number of quark flavours efectively produced at a given energy of interaction and N_C is the number of quark colour state.

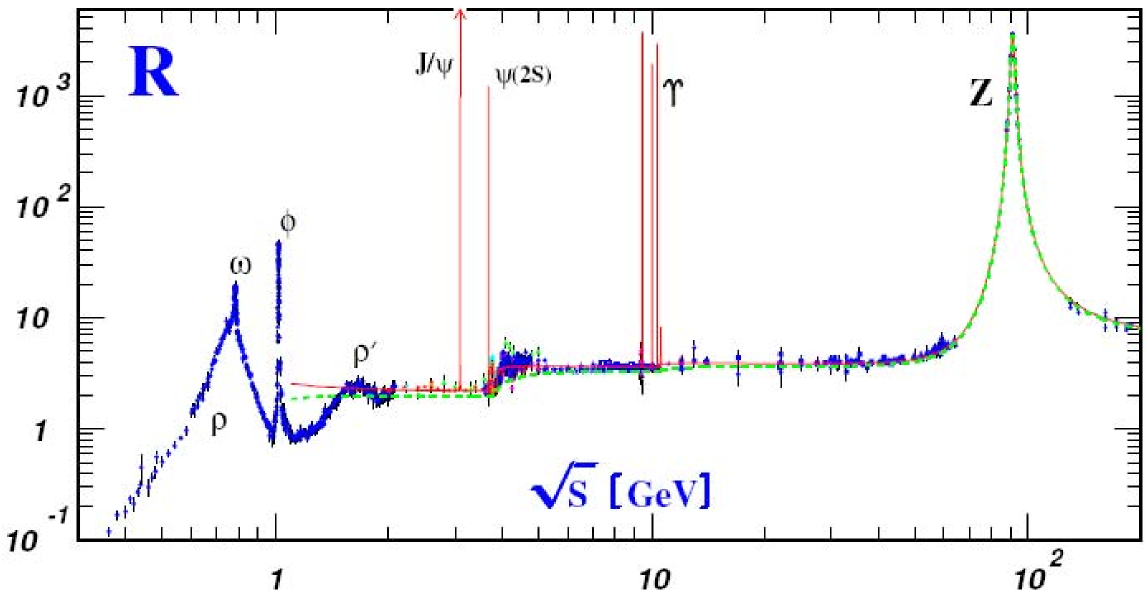


Fig. 5: Experimental values of the ratio $R_{e^+e^-}$ vs energy of e^+e^--beam.

Remark. For more realistic calculation of the ratio $R_{e^+e^-}$ it is inevitable to take into account also the higher orders of perturbative expansion, e.g. events of the type $e^+e^- \to q\bar{q}g$ (see Fig. 6).

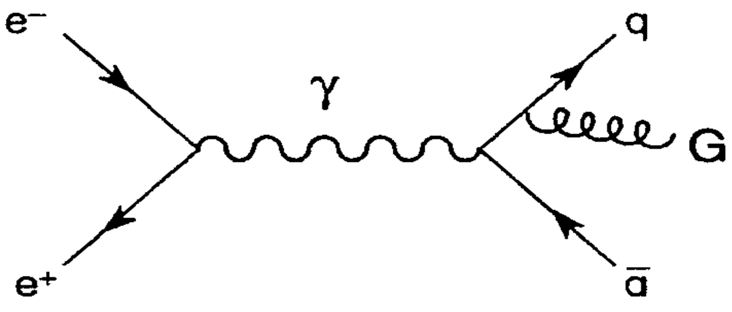
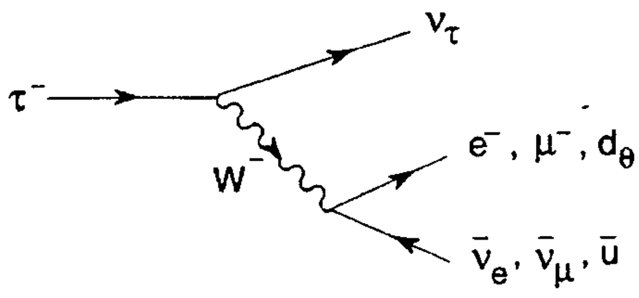


Fig. 6: Radiation of bremstrahlung gluon, the next order correction to the process $e^+e^- \to q\bar{q}$.

Decay of τ-lepton. This decay presents an additional evidence for N_C=3. The mentioned decay goes through W-emission (Fig. 7). As coupling of W-boson to weak charge currents is the same for all leptons and quarks, we have ($2+N_C$) equal contributions (if we neglect the masses of



Obr.7: Diagram of τ–lepton decay.

quarks and leptons and also the contribution of strong interactions). Two of them represent leptonic decay modes ($\tau^- \to \nu_\tau e^- \bar{\nu}_e$ a $\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu$) and additional N_C is connected with $q\bar{q}$ -pairs of different colors: $\tau^- \to \nu_\tau d_\theta u$ ($d_\theta = \cos\theta_C\, d + \sin\theta_C\, s$). As a consequence it is expecting:

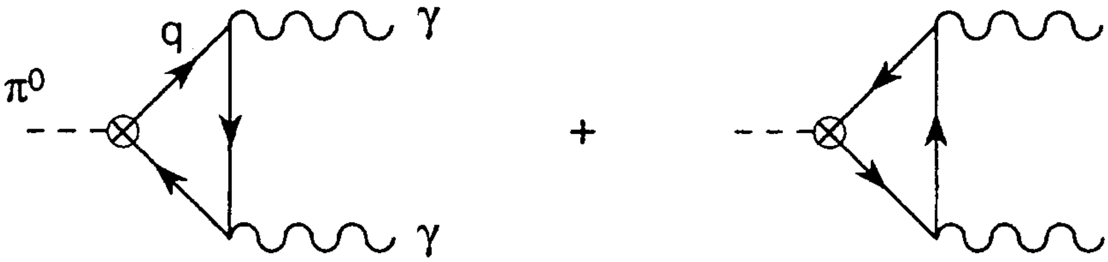
$$B_{\tau\to l} \equiv Br(\tau^- \to \nu_\tau l^- \bar{\nu}_l) \approx \frac{1}{2+N_C} = \frac{1}{5} = 20\%$$
$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + hadrons)}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} \tag{2.11}$$

At the same time experiment gives:

$$B_{\tau\to e} = (18.01 \pm 0.18)\% , \qquad B_{\tau\to\mu} = (17.65 \pm 0.24)\%$$
$$R_\tau = (1 - B_{\tau\to e} - B_{\tau\to\mu})/B_{\tau\to e} = 3.56 \pm 0.04 \tag{2.12}$$

It is in a quite good agreement with hypothesis N_C=3.

Decay of π^0-meson. The decay $\pi^0 \to \gamma\gamma$ goes through a triangle quark loop (see Fig. 8).



Obr.8: Triangle quark loops generating decay $\pi^0 \to \gamma\gamma$

The cross vertex denotes the axial current $A^3_\mu \equiv \left(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d\right)$. The direct calculation gives:

$$\Gamma\left(\pi^0 \to \gamma\gamma\right) = \left(\frac{N_C}{3}\right)^2 \frac{\alpha^2 m_\pi^3}{64\pi^3 f_\pi^2} = 7.73\ eV \qquad (2.13)$$

where $f_\pi = 92.4\ MeV$ is the coupling constant of π^0 to A^3_μ, which is obtained from $\pi^- \to \mu^- \tilde{\nu}_\mu$ (assuming the isospin symmetry). The agreement with the experimental value $\Gamma = 7.7 \pm 0.6$ is remarkable.

2.4 Strong interactions and symmetries

Strong interactions exhibit a series of symmetries:

- charge ***SU(3)**$_c$* symmetry
- symmetry of space inversion
- symmetry of charge conjugation
- „flavor“ ***SU(n)**$_f$* (*n=3,4,..*) symmetry.

Symmetry of interaction Lagrangian leads to conservation laws (*Noether theorem*). In case of strong interactions it leads to the following conservation laws:

- Space and charge parity
- isospin, strangeness, „charm“, „beauty“, „true“
- in strong interactions are valid also generally valid conservation laws connected with valid symmetries (the law of conservation of 4-momentum, angular momentum and charge).

The charge SU(3)$_c$ symmetry. From the view point of strong interactions this symmetry means that colored charges are equivalent.

2.5 Group SU(3).

Definition. It is a group of unitary and unimodular transformations in 3-dimensional complex space:

$$\{U, \quad U^{+}U = 1 \quad \text{a} \quad \det U = 1\} \tag{2.14}$$

This group has 8 free parameters $n = N_c - N_u - N_d$:

- $N_c=18$ is number of parameters of complex matrix 3×3.
- $N_u=9$ is number of conditions coming from unitarity
- $N_d=1$ is number of conditions coming from unimodularity

A general form of ***SU(3)*** element is**:**

$$\boldsymbol{U} = \boldsymbol{e}^{i\varepsilon_\alpha L\alpha} \quad \boldsymbol{\alpha} = 1\cdots 8 \tag{2.15}$$

where ε_α is 8 real parameters (angles of rotation in v C_3),

$\boldsymbol{L_\alpha}$ - independent hermitian operators ($\boldsymbol{L}_\alpha^+ = \boldsymbol{L}_\alpha$) with zero trace, which are the generators of infinitesimal transformation of the grupe SU(3):

$$\boldsymbol{U} = 1 + \boldsymbol{i}\boldsymbol{\varepsilon}_\alpha \boldsymbol{L}_\alpha \tag{2.16}$$

Commutation relations for L_α are

$$[\boldsymbol{L}_\alpha, \boldsymbol{L}_\beta] = \boldsymbol{f}_{\alpha\beta\gamma} \boldsymbol{L}^\gamma \quad \boldsymbol{\gamma} \equiv \text{sum. index} \tag{2.17}$$

where $f_{\alpha\beta\gamma}$ are anti-symmetric structural coefficients.

Two of the $\boldsymbol{L_\alpha}$ operators are diagonal (They cannot be 3 because $\boldsymbol{L_\alpha}$ has zero trace).

The Gell-Mann choice of $\boldsymbol{L_\alpha}$ for 3-dimensional representation of SU(3):

$$\boldsymbol{L}_\alpha = \frac{1}{2}\boldsymbol{\lambda}_\alpha \tag{2.18}$$

where

$$\boldsymbol{\lambda}_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \boldsymbol{\lambda}_2 = \begin{pmatrix} 0 & -\boldsymbol{i} & 0 \\ \boldsymbol{i} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \boldsymbol{\lambda}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\boldsymbol{\lambda}_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \boldsymbol{\lambda}_5 = \begin{pmatrix} 0 & 0 & -\boldsymbol{i} \\ 0 & 0 & 0 \\ \boldsymbol{i} & 0 & 0 \end{pmatrix} \quad \boldsymbol{\lambda}_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{2.19}$$

$$\boldsymbol{\lambda}_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\boldsymbol{i} \\ \boldsymbol{i} & 0 & 0 \end{pmatrix} \quad \boldsymbol{\lambda}_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Structural coefficients:

$$\begin{aligned} &\boldsymbol{f}_{123} = 1 \quad \boldsymbol{f}_{147} = -\boldsymbol{f}_{156} = \boldsymbol{f}_{246} = \boldsymbol{f}_{257} = \boldsymbol{f}_{345} = -\boldsymbol{f}_{367} = 1/2 \\ &\boldsymbol{f}_{458} = \boldsymbol{f}_{678} = \sqrt{3}/2 \quad \text{ostatné} \quad \boldsymbol{f}_{\alpha\beta\gamma} = 0 \end{aligned} \tag{2.20}$$

Representations of group SU(3).

Matematical approach.

Representation of group ***G*** in a vector (linear) space ***X*** we understand transformation ***T***, which assigns to each element ***g*** of group ***G*** a linear operator ***T(g)*** in space ***X***, in such a way that is valid:

1. $T(e) = \underline{1}$, where ***e*** is the unit of the group ***G*** and $\underline{1}$ is unit operator in space ***X***,
2. $T(g_1 \cdot g_2) = T(g_1) \cdot T(g_2)$ for all $g_1, g_2 \in G$

The space ***X*** is a space of representation, operators ***T(g)*** are operators of representation. If *dim(**X**)* = ***n***, then we speak about ***n–dimensional representation*** (or representation of n^{th}–degree). In such case the elements of group **G** we can represent in the space ***X*** by matrices $n \times n$.

Group $SU(3)_c$ Physical meaning.

Quark can be in 3 colored charge states:

$$|Q_1\rangle = |R\rangle \qquad |Q_2\rangle = |G\rangle \quad |Q_3\rangle = |B\rangle \tag{2.21}$$

The colored (charge) state of quark can be expresses as follows:

$$|q\rangle = q^1|R\rangle + q^2|G\rangle + q^3|B\rangle \rightarrow \begin{pmatrix} q^1 \\ q^2 \\ q^3 \end{pmatrix}$$
$$|q^1|^2 + |q^2|^2 + |q^3|^2 = 1 \tag{2.22}$$

where q^i , *i=1..3* are coordinates of vector $|q\rangle$ in 3-demensional complex space of quark colored state $\rightarrow$ their interpretation:

$|q^i|^2$ is probability of quark to be in colored state $|Q_i\rangle$.

The space of quark colored charge state $F_3(RGB)$, i.e. the space with the base $\left(|R\rangle, |G\rangle, |B\rangle\right)$ over the field of complex numbers, is isomorphic to the 3-dimensional complex space C_3, hence states from $F_3(RGB)$ can be represented by vectors from C_3 **:**

$$F_3(RGB) \xrightarrow[isomorfizmus]{} C_3 \tag{2.23}$$

Action of group $SU(3)_c$ ***in*** $F_3(RGB)$**.** From physical point of view group $SU(3)_c$ change color composition of colored charge state (changes proportion of colored components of charge state) , hence transforms colored state $|q\rangle \in F_3(RGB)$ to other colored state $|q'\rangle \in F_3(RGB)$:

$$|q\rangle = q^1|R\rangle + q^2|G\rangle + q^3|B\rangle \xrightarrow[g\in SU(3)_c]{} |q'\rangle = q'^1|R\rangle + q'^2|G\rangle + q'^3|B\rangle \tag{2.24}$$

In space C_3 (space of representation) an action of the element $g \in SU(3)_c$ will manifests as follows

$$\begin{pmatrix} q^1 \\ q^2 \\ q^3 \end{pmatrix} \xrightarrow[g\in SU(3)]{} \begin{pmatrix} q'^1 \\ q'^2 \\ q'^3 \end{pmatrix} = \underbrace{\exp\left(\frac{i}{2}\varepsilon_\alpha \lambda^\alpha\right)}_{T(g)} \begin{pmatrix} q^1 \\ q^2 \\ q^3 \end{pmatrix} \tag{2.25}$$

If a colored state of quark we express by means of a triplet of complex numbers (q^1, q^2, q^3), i.e. by means of a vector from C_3, then elements of the group $SU(3)_c$ will be represented in the space of representation (C_3) by complex matrices of 3×3 :

$$g \to T(g) = \exp\left(\frac{i}{2}\varepsilon_\alpha \cdot \lambda^\alpha\right), \quad \alpha = 1...8 \ , \tag{2.26}$$

where λ_α are the Gell-Mann matrices. In such a case we speak about the so called ***fundamental representation of the group*** $SU(3)_c$ **(3).**

Summary. At the fundamental representation of the group $SU(3)_c$ the space C_3 is the space of representation (hence the 3-dimensional color space $F_3(R,B,G)$) is represented by the 3-dimensional complex space C_3) and elements of the group $SU(3)_c$ are represented by complex matrices of 3×3.

The invariance of strong interactions to the group $SU(3)_c$ means that ***dynamics of colored quarks*** (interaction cross sections) does not depend on ***type of colored charge***.

Adjoint representation

Let us consider a quark in a colored state $|q\rangle$ and make the charge conjugation, i.e. let us change a particle by its anti-particle:

$$|q\rangle \xrightarrow{C} |\bar{q}\rangle = \bar{q}^1|\bar{R}\rangle + \bar{q}^2|\bar{G}\rangle + \bar{q}^3|\bar{B}\rangle \xrightarrow[\text{isomorfizmus do } C_3]{} \begin{pmatrix} \bar{q}^1 \\ \bar{q}^2 \\ \bar{q}^3 \end{pmatrix} \equiv \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \qquad (2.27)$$

$$q_i = \bar{q}^i \equiv \left(q^i\right)^*$$

where $|\bar{R}\rangle, |\bar{G}\rangle$ a $|\bar{B}\rangle$ are anti-colors to colors $|R\rangle, |G\rangle, |B\rangle$ and coordinates q_i we will call covariant coordinates of colored state (unlike to q^i – contravariant coordinate). The covariant coordinates of colored state represent by yourselves the coordinates of conjugated (hence „anti-colored“) state in the space $F_3(\bar{R},\bar{B},\bar{G})$ (3-dimensional colored space with the basis $\left(|\bar{R}\rangle, |\bar{G}\rangle, |\bar{B}\rangle\right)$ over the field of complex numbers – it is a dual space to the space $F_3(R,B,G)$.

It is evident that in the space **C₃** that represent the space of „anti-colors“ ($F_3(\bar{R},\bar{B},\bar{G})$ the elements of group $SU(3)_c$ are represented by the ***C***-matrices of ***3×3*** :

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \xrightarrow[g\in SU(3)]{} \begin{pmatrix} q'_1 \\ q'_2 \\ q'_3 \end{pmatrix} = \begin{pmatrix} q'^1 \\ q'^2 \\ q'^3 \end{pmatrix}^* = \underbrace{\exp\left(-\frac{i}{2}\varepsilon_\alpha \lambda^\alpha\right)}_{T(g)} \begin{pmatrix} q^1 \\ q^2 \\ q^3 \end{pmatrix}^* , \qquad (2.28)$$

that in general have the following form:

$$\hat{U} = \exp\left(-\frac{i}{2}\varepsilon_\alpha \cdot \lambda^{\alpha *}\right) = U^*$$
$$q_i \to q'_i = \left(U^i_{\ k}\right)^* q_k = \left(U^+\right)^k_i q_k \qquad (2.29)$$

In this case (action of $SU(3)_c$ in space of colored state of anti-quarks) we speak about the adjoint representation ***(*** $\bar{3}$ ***)***.

Representations of higher dimension.

From fundamental **(3)** and adjoint representation **(** $\overline{3}$ **)** it is possible to create representation of higher ranks. Physically it means that on the base of colored states of quark and anti-quark combinations.

The colored bi-combinations created from the colored state of quark ($|\boldsymbol{q}\rangle$) and anti-quark ($|\overline{\boldsymbol{q}}\rangle$), which we will denote as $|\boldsymbol{q\overline{q}}\rangle$, represent by themselves a vector v C-space with the basis $|\boldsymbol{Q}_i\rangle \otimes |\overline{\boldsymbol{Q}}_j\rangle$ $\boldsymbol{i, j = 1...3}$ and a bi-colored state is represented by coordinates $\boldsymbol{T^i_j = q^i q_j}$, i.e. by tensor of the group SU(3). The space of bi-colored combination is a tensor product of one-component colored spaces, for which is valid:

$$3 \otimes \overline{3} = 1 \oplus 8 \qquad (2.30)\ .$$

The decomposition (2.30) means, that 9-dimensional space of bi-colored state it is possible to decompose into two enclosed (from the view point of the SU(3) group) sub-spaces: the subspace of colored singlets and subspace of colored octets.

Basis of the SU(3) singlet: $\frac{1}{\sqrt{3}}\left(|R\overline{R}\rangle + |B\overline{B}\rangle + |G\overline{G}\rangle\right)$

Basis of the SU(3) octet:

$$|R\overline{G}\rangle \quad |R\overline{B}\rangle \quad |G\overline{R}\rangle \quad |G\overline{B}\rangle \quad |B\overline{G}\rangle \quad |B\overline{R}\rangle$$

$$\frac{1}{\sqrt{2}}\left(|R\overline{R}\rangle - |G\overline{G}\rangle\right) \quad \frac{1}{\sqrt{6}}\left(|R\overline{R}\rangle + |G\overline{G}\rangle - 2|B\overline{B}\rangle\right)$$

In nature the colored singlet is realized by **mesons** ($\boldsymbol{q\overline{q}}$) and the colored octet by **gluons.**

Tri-colored combinations. The colored space is can be decomposed as follows:

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$$

In nature occurs the colored singlet combination that is realized by ***baryons***.