3 Hadron classification

3.1 Isotopic SU(3) symetry.

The year 1947: the discovery of π–meson (Yukawa's particle) – it is a beginning of the discovery of a whole series of strongly interacting particle (π–mesons, K–mesons, Σ– hyperons, etc.). Up to that time had been known only nucleons as strongly interacting particles (hadrons).
A big number of the discovered particles invoked a question how to categorize them.

Quark hypothesis (Gell-Mann, Zweig):

There exists a fundamental triplet of particles – quarks from which can be created all hadrons. The three particle states representing quarks ***u, d*** and ***s*** create the basis of fundamental representation of group SU(3).
The physical meaning: quark can be in 3 isotopic states that from the point of view of strong interaction are equivalent, i.e. dynamic of strong processes do not change if the type quark is change.

Isospin and hypercharge (quantum numbers I_3 and Y)

Two of the Gell-Mann matrices (λ_i , i=1..8) are diagonal λ_3 a λ_8 therefore the quarks *u, d* and *s* can be taken as eigenstates. On the base of λ_3 and λ_8 are defined operators of ***isospin*** (its 3rd components) $\boldsymbol{I_3}$ and ***hypercharge Y:***

$$I_3 = \frac{1}{2}\lambda_3 \qquad Y = \frac{1}{\sqrt{3}}\lambda_8 \quad . \tag{1}$$

The quarks *u, d* and *s* are the eigenstates of operator $\boldsymbol{I_3}$ and ***Y*** and can be characterized by eigenvalues of these operators. Taking into account that:

$$u = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad d = \begin{pmatrix}0\\1\\0\end{pmatrix} \quad s = \begin{pmatrix}0\\0\\1\end{pmatrix} \qquad I_3 = \frac{1}{2}\begin{pmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & 0\end{pmatrix} \quad Y = \frac{1}{3}\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & -2\end{pmatrix}, \tag{2}$$

Then in the plane (I_3, Y) the quarks *u*, *d* and *s* and the corresponding anti-quarks will be depicted as follows:

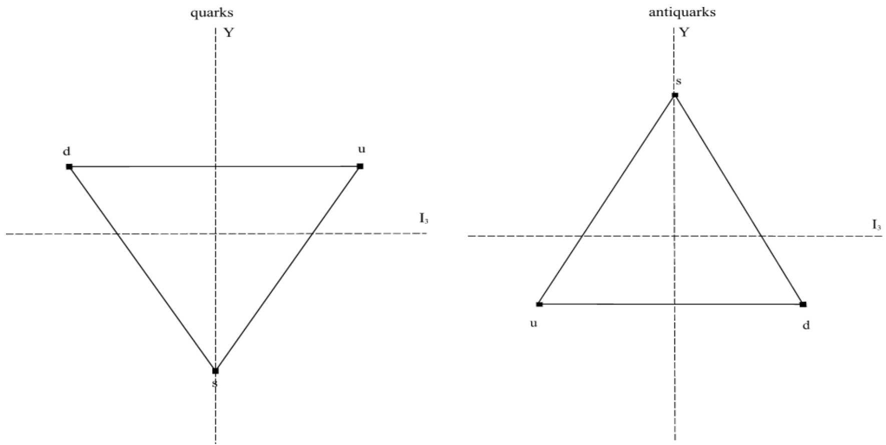

Figure 1: Quarks and anti-quarks in the plane (I_3. Y)

3.2 Mesons (system $q\bar{q}$).

Mesons are strongly interacting particles that consist of quark and anti-quark. Let us have $q\bar{q}$ -pár made of (*u*, *d*, *s*). The rules of composing of the SU(3) representations leads to the following:

$$3\otimes\bar{3}=1\oplus 8$$

Hence $q\bar{q}$ - pairs belong to the SU(3) singlet or SU(3) octet:

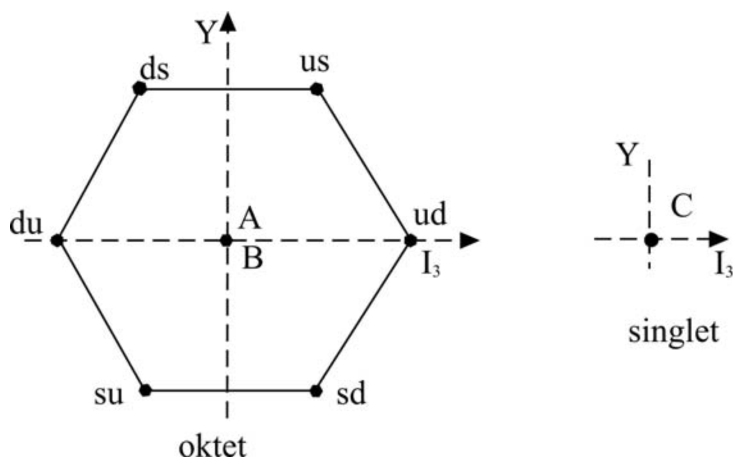

Figure 2:The maeson octet and singlet, the used quarks are *u*,*d* and *s*.

$$A=\frac{1}{\sqrt{2}}\left(u\bar{u}-d\bar{d}\right)\quad C=\frac{1}{\sqrt{3}}\left(u\bar{u}+d\bar{d}+s\bar{s}\right)\quad B=\frac{1}{\sqrt{6}}\left(u\bar{u}+d\bar{d}-2s\bar{s}\right)$$

3.2.1 Basic characteristics of $q\bar{q}$ system.

The basic characteristics of the bound $q\bar{q}$ system are summarized below:

- Discrete spectrum of energy levels.
- Spin $q\bar{q}$ - pair:
  1. The internal spin: S= 0, 1 (adding of 2 spins of 1/2).
  2. The total spin: $\vec{J} = \vec{L} + \vec{S}$, where $\vec{L}$ is orbital momentum and $\vec{S}$ is internal spin.
- Parity of $q\bar{q}$ - pair: $P = -(-1)^L$; sign $-$ before bracket corresponds opposite parity of quark and anti-quark and $(-1)^L$ is the parity of spherical function $Y_{LM}(\theta,\varphi)$.
- Charge parity (in case of neutral $q\bar{q}$ - pairs): $C = (-1)^{L+S}$ (eigenstates of operator of charge conjugation: the replacement $q \leftrightarrow \bar{q}$, exchange of positions and spins of the quarks)
- Characteristics of $q\bar{q}$ - pair: J^{PC}

3.2.3 Scalar mesons (J=0)

Table 1: The octet and singlet of scalar mesons, the mesons are made of *u*, *d* and *s*.

	octet								*singlet*
Meson	π^+	π^0	π^-	K^+	K^0	K^0	K^-	η_8	η_1
Structure	$(u\bar{d})$	$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$	$(d\bar{u})$	$(u\bar{s})$	$(d\bar{s})$	$(s\bar{d})$	$(s\bar{u})$	$\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$	$\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} + s\bar{s})$
m[MeV]	139.57	134.98	139.57	493.68	497.65		493.68	957.78 (η')	547.75 (η)
τ[ns]	26.03	$8.4\ 10^{-8}$	26.03	12.38	0.0895 / 51.8		12.38	-	-
I	1			1/2		1/2		0	0
Y	0			-1		1		0	0

$I \equiv$ isospin , $Y \equiv$ hypercharge

Mixing of neutral mesons

Due to violation of the SU(3)- (flavor) symmetry the fully neutral mesons η_1 and η_8 could undergo mixing and the physically observable are the mesons η and η' that can be expressed through a mixing angle θ_P :

$$\eta = \eta_8 \cos\theta_P - \eta_1 \sin\theta_P \qquad \eta' = \eta_8 \cos\theta_P + \eta_1 \sin\theta_P \ .$$

The neutral kaons $\boldsymbol{K^0}$ and $\boldsymbol{\bar{K}^0}$ differs by their hypercharge (-1 and 1, respectively). Hypercharge is conserved in strong and electromagnetic interactions but it is not conserved in weak interactions. Due to the weak interactions are possible the transitions $\boldsymbol{K^0 \leftrightarrow \bar{K}^0}$, hence the $\boldsymbol{K^0\,\bar{K}^0}$ –oscillations. The eigenstates of full Hamiltonian are certain combination $\boldsymbol{K^0}$ and $\boldsymbol{\bar{K}^0}$:

$$K_L^0 = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\varepsilon\, K_1^0 + K_2^0\right) \qquad K_S^0 = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(K_1^0 + \varepsilon\, K_2^0\right)$$

where $\boldsymbol{K_1^0 = \frac{1}{\sqrt{2}}\left(K^0 + \bar{K}^0\right)}$ and $\boldsymbol{K_2^0 = \frac{1}{\sqrt{2}}\left(K^0 - \bar{K}^0\right)}$ are eigenstates of the CP operator (combined charge and space parity), ε is a small ($|\varepsilon| << 1$) quantity. If $\varepsilon \neq 0$ it means that in weak interactions neither the CP-parity is not conserved.

Consequence: the physically well defined mass have $\boldsymbol{K_L^0}$ and $\boldsymbol{K_S^0}$ and mass difference Δm = $m(K_L^0) - m(K_L^0)$ = (0.5333±0.0027)×10^{10} hs^{-1} defined the oscillations of the system $\boldsymbol{K^0\,\bar{K}^0}$.

3.2.4 Vector mesons (J=1)

Table 2: The vector meson octet and singlet, the mesons are made of the quarks *u*, *d* and *s*.

	Octet								*singlet*
meson	ρ^+	ρ^0	ρ^-	K^{*+}	K^{*0}	K^{*0}	K^{*-}	$\eta^*{}_8$	$\eta^*{}_1$
structure	$(u\bar{d})$	$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$	$(d\bar{u})$	$(u\bar{s})$	$(d\,\bar{s})$	$(s\,\bar{d})$	$(s\bar{u})$	$\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$	$\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} + s\bar{s})$
I	1			1/2		1/2		0	0
Y	0			-1		1		0	0

Experimentally are not observed the $\boldsymbol{\eta^*{}_1}$ and $\boldsymbol{\eta^*{}_8}$ but their combinations – the mesons $\boldsymbol{\varphi}$ and $\boldsymbol{\omega}$:

$$\boldsymbol{\varphi = s\bar{s}} \quad \text{and} \quad \boldsymbol{\omega \approx \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right)} \quad .$$

3.3 Baryons. Systems qqq

Baryons are 3-quarks systems that are the color singlet hence their state vector Ψ_{color} describing the colored charge state is *anti-symmetric*.

The full state vector (wave function) is constructed as follows:

$$\underbrace{\Psi_{full}}_{(-)} = \underbrace{\Psi_{color}}_{(-)} \cdot \underbrace{\Psi_{spin} \cdot \Psi_{iso}}_{(+)} \cdot \underbrace{\Psi_{space}}_{(+)}$$

The signs below the individual components of Ψ give their symmetry. The component Ψ_{space} is symmetric as we assume that the system of quarks is in the state with the orbital quantum number L = 0. The full wave function Ψ_{full} must be anti-symmetric because it is a system of identical fermions.

Isotopic structure. Let us assume that there are three types of quark (*u*, *d* and *s*) then structure of the isotopic part of wave function (Ψ_{iso}) is given by the decomposition:

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

Symmetry: S M_S M_A A

S≡ the isotopic part of wave function is fully symmetric;

M_S (M_A) ≡ the isotopic part of wave function is (anti-)symmetric in two first indexes;

A ≡ the isotopic part of wave function is fully anti-symmetric.

Example: The system composed of quarks *u*, *u* and *d*.*:*

$$\Delta = \frac{1}{\sqrt{3}}(uud + udu + duu)$$

$$p_A = \frac{1}{\sqrt{2}}(ud - du)u$$

$$p_S = \frac{1}{\sqrt{3}}(udu + duu - 2uud)$$

Δ is symmetric;

p_A has a mixed symmetry – only the first two indexes are anti-symmetric;

p_S has a mixed symmetry – the first two indexes are symmetric.

Spin structure. The structure of spin part of wave function (Ψ_{spin}) is given by the decomposition:

$$2 \otimes 2 \otimes 2 = (4 \oplus 2) \otimes 2 = 4 \oplus 2 \oplus 2$$

The symmetry → S M_S M_A

The symmetry of product of spin and isospin components of wave function leads to the following categorization of the (SU(3), SU(2))–multiplets:

S	:	(10, 4) + (8, 2)
M_S	:	(10, 2) + (8,4) + (8, 2) + (1, 2)
M_A	:	(10, 2) + (8,4) + (8, 2) + (1, 2)
A	:	(1, 4) + (8, 2)

We are interested in the symmetric states. Symmetry of the decuplet is obvious and the symmetric octet arises as follows:

$$\frac{1}{\sqrt{2}}\left[\left(\underset{M_S}{8}\,,\ \underset{M_S}{2}\right) + \left(\underset{M_A}{8}\,,\ \underset{M_A}{2}\right) \right]$$

The experiment shows that baryons with lower masses are in octuplet with spin 1/2 and dekuplet with spin 3/2 .

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3.4 Quark systems with *c* and *b* quarks

If into the scheme with quarks *u*, *d* and *s* we add quark *c* then the flavor symmetry will be spread from SU(3) to SU(4). Inclusion of the quark *b* leads to the symmetry SU(5).

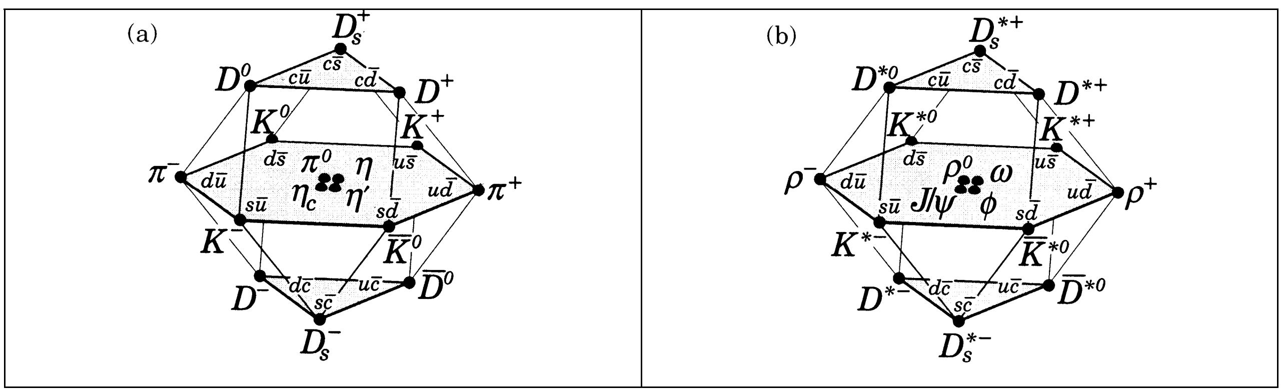

Figure 3: The SU(4) 16-plets for the pseudo-scalar (a) and (b) vector mesons made of *u*, *d*, *s* and *c* quarks.

Mesons in the SU(4)-scheme. In this case building quarks are *u*, *d*, *s* and *c* and basic multiplets are the 16-plets of scalar and vector mesons (see Figure 3). At the same time the nonet of light mesons is in central plane.

Table 3: The charmed scalar mesons, the mesons are made of *u, d, s* and *d*. $\tau \equiv$ mean lifetime; $m \equiv$ mass of meson

Scalar charmed mesons						
meson	D^+	D^0	$\overline{D}^0$	D^-	D_s^+	D_s^-
structure	$c\bar{d}$	$c\bar{u}$	$\bar{c}u$	$\bar{c}d$	$c\bar{s}$	$\bar{c}s$
cτ[μm]	311.8 ±2.1	123.0±0.4		311.8 ±2.1	149.9 ±2.1	
m[MeV]	1869.3±0.4	1864.5±0.4		1869.3±0.4	1968.2±0.4	

$\boldsymbol{D^0\overline{D}^0}$ ***mixing***. The mesons $\boldsymbol{D^0}$ and $\boldsymbol{\overline{D}^0}$ are not eigenstates of the full Hamiltonian therefore (analogically as in case of the neutral kaons) there can occur the transitions $\boldsymbol{D}^0 \leftrightarrow \boldsymbol{\overline{D}}^0$ (oscillations of $\boldsymbol{D^0\overline{D}^0}$). The physical states with well defined masses are D_1^0 and D_2^0. The size of these oscillations is however small.

System $\boldsymbol{c\bar{c}}$ **.** This system presents a special group of mesons with hidden charm. The lightest representatives of the system are η_c (state: $1\,^1S_0$, m=2979.8 MeV) and J/ψ ($1\,^3S_1$, m=3096.88 MeV). However we also know the higher excited states (1P: *h(1P)*, $\chi_{c0}(1P)$ $\chi_{c1}(1P)$ $\chi_{c2}(1P)$, 1D: *ψ(3770), etc.).*

Baryons in the SU(4)-scheme

At enhancement of the symmetry from SU(3) to SU(4) (including of c-quark) the baryon octet and decuplet transform to the (SU(4)–) 20–plet (see Figure). Each SU(4) multiplet is characterized by the same spin and parity.

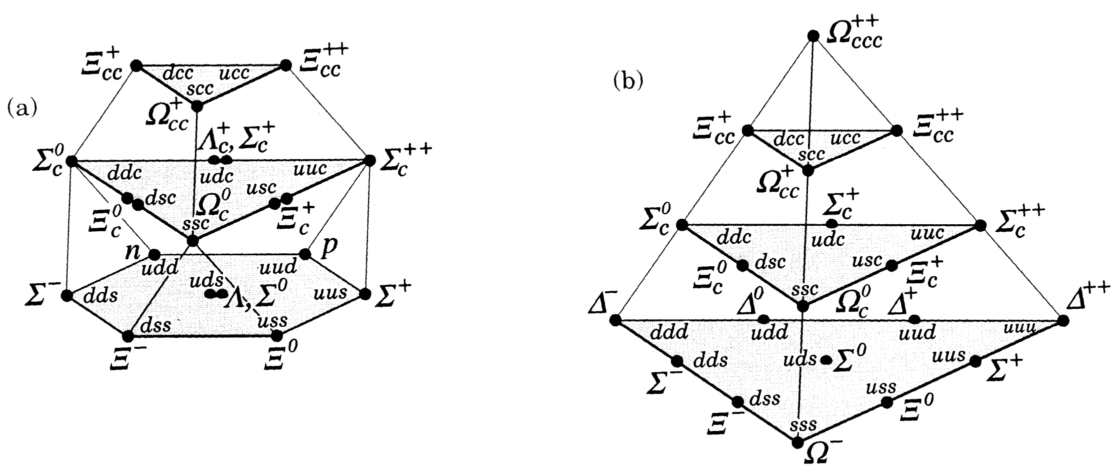

Figure 4: The SU(4) multiplets of baryons made of *u, d, s* and *c* quarks: the 20-plet with the SU(3) octet (a) and with the SU(3) decuplet (b).

B–mesons

To this group belong the mesons containing the *b*–quark. From the view point of the standard model (SM) they are very important as they enable to test the SM in many points (CP–violation, perturbative QCD, etc.).

Table 4: The stable B-mesons.

Stable b–mesons								
Meson	B^+	B^0	$\overline{B}_0$	B^-	B_s^0	$\overline{B}_s^0$	$\boldsymbol{B_c^+}$	$\boldsymbol{B_c^-}$
structure	$u\overline{b}$	$d\overline{b}$	$\overline{d}b$	$\overline{u}b$	$s\overline{b}$	$\overline{s}b$	$\boldsymbol{c\overline{b}}$	$\boldsymbol{\overline{c}b}$
$\boldsymbol{c\tau}$[μm]	491.1±3.3	458.7±2.7		491.1±3.3	431.8 ± 9.3		137.9 ± 21.	
$\boldsymbol{m}$[MeV]	5279.1±0.4	5279.5 ± 0.5		5279.1±0.4	5366.4 ± 1.1		6286. ± 50.	

$B^0\overline{B}^0$ -mixing. The mesons $\boldsymbol{B^0}$ and $\boldsymbol{\overline{B}^0}$ as well as $\boldsymbol{B^0_S}$ and $\boldsymbol{\overline{B}^0_s}$ are not (analogically as in case of D-mesons) the eigenstates of the full Hamiltonian therefore the transitions of $\boldsymbol{B_0} \leftrightarrow \boldsymbol{\overline{B}_0}$ (oscillations of $\boldsymbol{B^0\overline{B}^0}$) are possible. The physical states with the well defined masses are $\boldsymbol{B_1^0}$ and $\boldsymbol{B_2^0}$. The size of the oscillation is big mainly for the $\boldsymbol{B^0_S}$–meson.

System $b\overline{b}$ **.** There exist a whole series of the $b\overline{b}$ -states (Ypsilonium):

Υ(1S), χ_{b0}(1P), χ_{b1}(1P), χ_{b2}(1P), Υ(2S), χ_{b2}(2S) …

For the physics of *b*-quark is significant first of all *Υ(1S)* with mass $m=9460.37 \pm 0.21$ *MeV*. Some of the existing $e^+ e^-$ colliders are tuned on resonance production of *Υ(1S)*, i.e. their collision energy in CMS is equal to the mass of *Υ(1S)* (KEKB (Belle), PEP II (BaBar)).

<u>Goals of B–physics:</u>

- Measurement with an increased accuracy of the SM parameters.
- To look for a possible evidence of a new physics (physics behind the SM)

Basic problems:

- The cross sections of b-quark production (tests of QCD)
- Determination of the parameters of the CKM (Cabibbo, Kobayashi, Maskawa) matrix.
- Spectroscopy of B–particles (confinement).

- $B^0\bar{B}^0$ mixing - the precise determination of the mixing parameters, especially in case of B^0_S and $\bar{B}^0_S$ mixing (looking for a new physics).
- Violation of the CP–symmetry.
- Measurement of the lifetimes of different B-hadrons (dynamics of b-decays, contribution of different diagrams, see Figure 5).

Appendix: The decay diagrams of B-meson

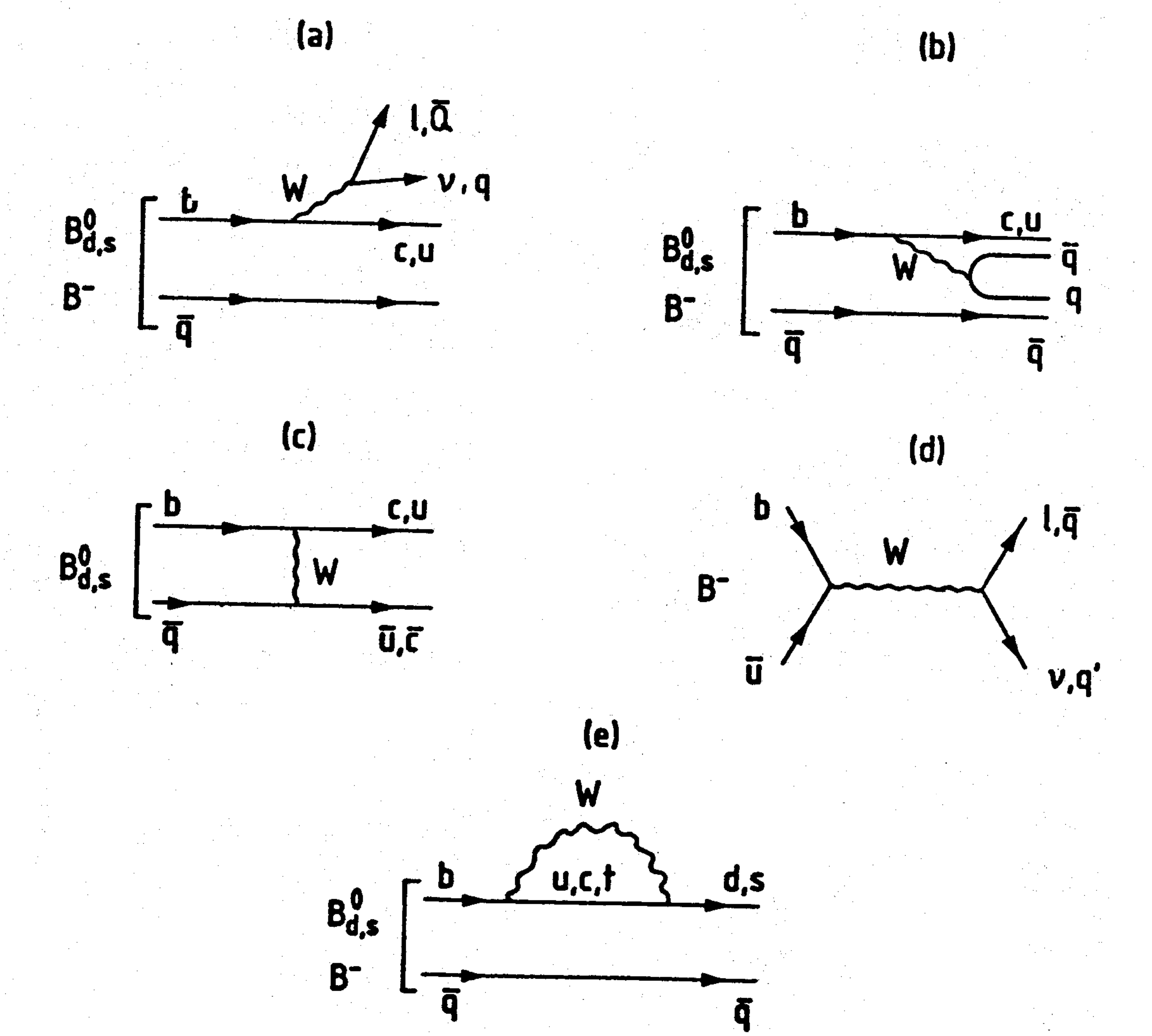

Figure 5: Diagrams of B-decays: (a) spectator diagram (light quark does not participate in the decay) – the main decay, (b) diagram with production of quark pair, (c) W-exchange in t-channel, (d) Annihilation (W-exchange in s-channel), (e) penguin-diagram.