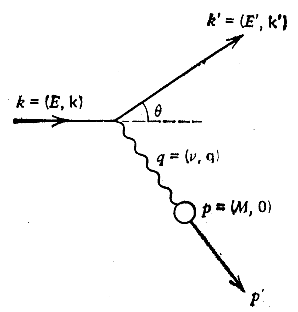
Structure of hadrons

For the scattering of $e\mu \rightarrow e\mu$ we have received:



The square of module of amplitude M_{fi} reads:

$$\overline{\left|M_{fi}\right|^2} = \frac{8e^4}{q^4} 2M^2 EE' \left\{ \cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2} \right\} \tag{1}$$

For the cross sections we have:

$$\frac{d\sigma}{dE'd\Omega} = \frac{(2\alpha E')^2}{q^4} \left\{ \cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2} \right\} \cdot \delta\left(\nu + \frac{q^2}{2M} \right) \tag{2}$$

where $\alpha = e^2/4\pi$, $\nu = E - E'$.

If we are interested only in scattering angle of electron:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}} \frac{E'}{E} \cdot \left\{ \cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2} \right\} \tag{3}$$

In general for scattering on non-point object one can write:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{point} \cdot \left| F(\vec{q}) \right|^2 \tag{4}$$

where $F(\vec{q})$ is the formfactor reflecting non-point structure of scattering center. For it is valid:

- At $|\vec{q}| \rightarrow 0 \quad \Rightarrow \quad F(0) = 1$ (wave length of intermediating virtual photon is much bigger than dimension of scattering center)
- $F(\vec{q}) = \int d^3\vec{x}\, \rho(\vec{x}) \exp(i\vec{q}\cdot\vec{x})$ is Fourier's picture of scattering center charge distribution.

Remark. From Maxwell equation for static potential ($A_\mu(x) = (\varphi(\vec{x}), 0)$) :

$\nabla^2\varphi(\vec{x}) = -Ze\rho(\vec{x})$ it follows $\int d^3\vec{x}\, e^{i\vec{q}\cdot\vec{x}}\nabla^2\varphi(\vec{x}) = -Ze\int d^3\vec{x}\, e^{i\vec{q}\cdot\vec{x}}\rho(\vec{x})$ and from here we get:

$$\int d^3\vec{x}\, e^{i\vec{q}\cdot\vec{x}}\varphi(\vec{x}) = \frac{Ze}{|\vec{q}|^2}\int d^3\vec{x}\, e^{i\vec{q}\cdot\vec{x}}\rho(\vec{x}) = \frac{Ze}{|\vec{q}|^2}F(\vec{q}).$$

On the other hand the amplitude of the electron scattering on point-like charge (see Chapter 8) has the structure:

$-iM = ie\bar{u}_f\gamma^0 u_i \cdot Ze/|\vec{q}|^2$, hence Fourier's picture of potential generated by non-point scattering center ($F(\vec{q})\, Ze/|\vec{q}|^2$) differs from the case of point-like center by the factor $F(\vec{q})$.

Let us assume the spherical symmetry of charge density $\rho = \rho(r),\ \ (r = |\vec{x}|)$ and we expand the formfactor into Taylor's series, we get:

$$F(\vec{q}) = \int d^3\vec{x}\,\rho(\vec{x})\left(1 + i\vec{q}\cdot\vec{x} - \frac{(\vec{q}\cdot\vec{x})^2}{6} + \cdots\right) = 1 - \frac{|\vec{q}|^2}{6}\langle r^2\rangle + \cdots \qquad (5)$$

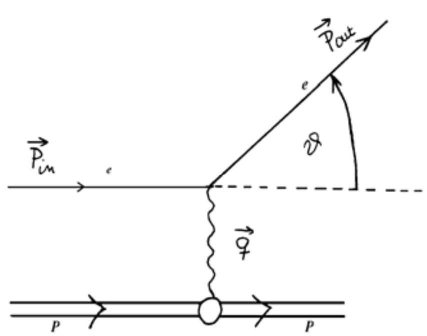
If we assume that $\rho(r) \sim \exp(-\lambda r)$ then

$$F(\vec{q}) \sim \frac{1}{\left(1 + \frac{|\vec{q}|^2}{\lambda^2}\right)} \qquad (6)$$

Hence the scattering center is characterized by space dimension 1/λ.

Electron-proton scattering

A) Amplitude of elastic *ep*-scattering



$$T_{fi} = i\int d^4x\, j_\mu\left(-\frac{1}{q^2}\right)J^\mu \qquad (7)$$

where $q = k' - k,\ j^\mu$ and J^μ are electrn and proton currents connected with transition of these particles from initial to final state :

$$j^\mu = -e\bar{u}(k')\gamma^\mu u(k)\exp(i(k-k')x) \qquad (8)$$

$$J^\mu = -e\bar{u}(p')\Gamma^\mu u(p)\exp(i(p-p')x) \qquad (9)$$

where the vertex function Γ appearing in the proton current can be expressed as follow:

$$\Gamma^{\mu} = F_1(q^2)\gamma^{\mu} + F_2(q^2)\frac{\kappa}{2m}\sigma^{\mu\nu}q_{\nu} \tag{10}$$

where F_1 and F_2 are independent formfactors κ is the anomalous magnetic moment proton.

Why Γ^{μ} has the strukture (10)?

- We showed that space spread of charge leads to formfactor (4) meaning that factor at γ^{μ} is changed: $1 \to F(|\vec{q}|)$.
- In addition to that at the ***eZe*** scattering, in 2nd order of perturbative expansion, contains ***eeγ***-vertex loop hence it exhibits a space structure. To this fact corresponds the replacement $\gamma^{\mu} \to \gamma^{\mu} + (\kappa/2m)\sigma^{\mu\nu}q_{\nu}$.
- At long-waved photon ($q^2 \to 0$) the vertex function Γ^{μ} must have a structure corresponding to point-like particle: $F_1(0) = 1$ and $F_2(0) = 0$.

If in expression for scattering amplitude we make the replacement $\gamma^{\mu} \to \Gamma^{\mu}$ and repeat the same procedure as at the ***eμ***-scattering, then instead of relation (1) we get:

$$\left.\frac{d\sigma}{d\Omega}\right|_{LS} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\left\{\left(F_1^2 - \frac{\kappa^2 q^2}{4M^2}F_2^2\right)\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}(F_1 - \kappa F_2)^2\sin^2\frac{\theta}{2}\right\} \tag{11}$$

Instead of F_1 and F_2 there are often used their combinations

$$G_E = F_1 + \frac{\kappa q^2}{4M^2}F_2 \qquad \text{and} \qquad G_M = F_1 + \kappa F_2 \tag{12}$$

called magnetic (G_M) and electric (G_E) formfaktor. An advantage of this expression is in the fact that in this case there is no interference term $G_E.G_M$:

$$\left.\frac{d\sigma}{d\Omega}\right|_{LS} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\left\{\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} - 2\tau G_M^2\sin^2\frac{\theta}{2}\right\} \tag{13}$$

where $\tau = -q^2/(4M^2)$.

Measurement of the $ep \to ep$ scattering cross section provides information about G_E and G_M – the experiment gives:

$$G_E(q^2) = \left(1 - \frac{q^2}{0.71}\right)^{-2} \quad \text{a} \quad G_M = \mu G_E \tag{14}$$

From the shape of $G_E(q^2)$ for the radius of proton we get:

An analogical radius (~***0.8 fm***) was found for distribution of magnetic moment.

Inelastic ep-scattering

At large Q^2 $(= -q^2)$ there is a big probability of proton fragmentation → it can be excited to Δ–resonance: $ep \rightarrow e\Delta^+ \rightarrow e\, p\, \pi^0$.

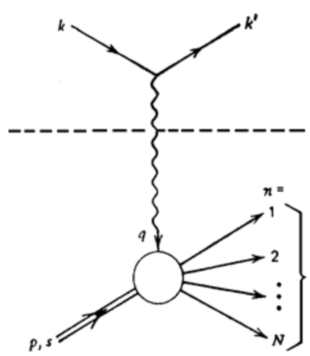


Fig. 1: Inelastic ep-scattering

Sach events can be characterized by

Invariant mass $W^2 = M_\Delta^2$.

Invariant mass is determined by the method of „missing mass":

$$W^2 = (p_1 + \cdots + p_N)^2 = (k + p - k')^2 \tag{16}$$

Cross section. In analogy with the $e\mu$–scattering for the differential cross section we can write:

$$\frac{d^2\sigma}{dE' d\Omega} = \frac{\alpha^2}{Q^4}\frac{E'}{E}|M|^2 = \frac{\alpha^2}{Q^4}\frac{E'}{E} L_{\mu\nu}^{(e)} W^{\mu\nu} \tag{17}$$

As we do not know the hadronic current for finding $W^{\mu\nu}$ we use phenomenology i.e. the conservation laws and independent momenta p and q. If intermediating particle is photon (no violation of C,P) then the most general form $W^{\mu\nu}$ is :

Comment [ST1]: In case when exchanged particle is W/Z-boson (case of interaction neutríno) will be present also the term W_3 violating C,P.

$$W^{\mu\nu} = W_1 g^{\mu\nu} + W_2 \frac{p^\mu p^\nu}{M^2} + W_4 \frac{q^\mu q^\nu}{M^2} + W_5 \frac{p^\mu q^\nu + q^\mu p^\nu}{M^2} \tag{18}$$

As the $L_{\mu\nu}$ is symmetric the contribution from the $W^{\mu\nu}$ gives only the symmetric part. The law of current conservation ($\partial_\mu J^\mu = 0$) leads to 2 conditions for the $W^{\mu\nu}$:

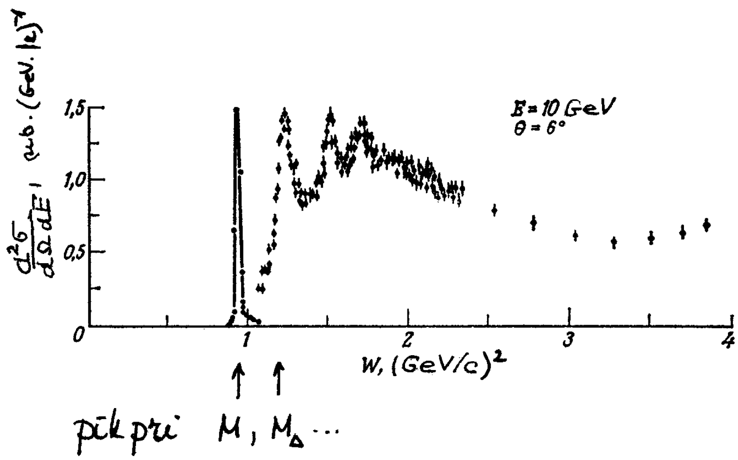


Fig. 2: The cross section of ep-scattering vs. invariant mass of proton fragments.

$$q_\mu W^{\mu\nu} = 0 \quad \text{a} \quad q_\nu W^{\mu\nu} = 0 \tag{19}$$

This means that only 2 from 4 W_i parameters are independent, the $W^{\mu\nu}$ can be parametrized:

$$W^{\mu\nu} = W_1\left(-g_{\mu\nu} + \frac{q^\mu q^\nu}{M^2}\right) + \frac{W_2}{M^2}\left(p^\mu - \frac{pq}{q^2}q^\mu\right)\left(p^\nu - \frac{pq}{q^2}q^\nu\right) \tag{20}$$

Where W_1 and W_2 are function scalar variables created from the 4-vectors in hadron vertex. There exist 2 independent variables: q^2 a $\nu = \frac{pq}{M}$. By means of these variables we can express the invariant mass of system of final hadrons:

$$W^2 = (p+q)^2 = M^2 + 2M\nu + q^2 \tag{21}$$

Remark. The additional kinematic variables that are used are:

$$x = -\frac{q^2}{2pq} \quad \text{and} \quad y = \frac{pq}{pk} \tag{22}$$

The cross section for inelastic ***ep***-scattering we get from the ***eμ***-scattering by the replacement $L^{(m)}_{\mu\nu} \to W_{\mu\nu}$:

$$\frac{d\sigma}{dE'd\Omega} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\left\{W_2(\nu,q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,q^2)\sin^2\frac{\theta}{2}\right\} \tag{23}$$

Scattering of virtual photon on parton

Finally the task of electron beam is that it ser ves as a source of virtual photons. The formulae for the total cross section of process of scattering of real photon on proton, $\gamma p \to X$, is similar to the formula (17):

$$\sigma^{\lambda}\left(\gamma p \to X\right) = \frac{4\pi^2\alpha^2}{K}\varepsilon_{\mu}^{\lambda *}\varepsilon_{\nu}^{\lambda} W^{\mu\nu} \tag{24}$$

where in case of real photon $K = \nu = q_0$ and we need to sum through 2 transverse polarization of photon (λ) in initial state and the current factor is $4MK$ (where $K=\nu$). In case of virtual photon ($q^2 \neq 0$) - its state is not limited by 2 transverse polarization and the current is arbitrary (current is well defined only in the case when particle is on its mass shell, hence fulfills corresponding equation of motion). At the same time the K can be chosen in such a way that it is valid:

$$W^2 = \left(p+q\right)^2 = M^2 + 2MK \quad \Rightarrow \quad K = \frac{W^2 - M^2}{2M} = \nu + \frac{q^2}{2M} \tag{25}$$

If we choose momentum of photon, $\vec{q}$, in the direction of axis z then for the vector of polarization of virtual photon we get:

$$\lambda = \pm 1: \quad \varepsilon_{\pm} = \mp\left(0,\, 1,\, \pm i,\, 0\right) \tag{26}$$

$$\lambda = 0: \quad \varepsilon_0 = \frac{1}{\sqrt{-q^2}}\left(\sqrt{\nu^2 - q^2},\, 0,\, 0,\, \nu\right) \tag{27}$$

For the total cross sections of absorption of transverse (σ_T) and longitudinal (σ_L) virtual photons we get:

$$\sigma_T = \frac{1}{2}\left(\sigma_+^{tot} + \sigma_-^{tot}\right) = \sigma_0 W_1\left(\nu, q^2\right) \tag{28}$$

$$\sigma_L = \sigma_0^{tot} = \sigma_0\left[\left(1 - \frac{\nu^2}{q^2}\right)W_2\left(\nu, q^2\right) - W_1\left(\nu, q^2\right)\right] \tag{29}$$

where $\sigma_0 = \frac{4\pi^2}{K}$. Hence we see that:

The structure functions W_1 and W_2 are unambiguously connected with the cross sections of absorption of virtual transverse and longitudinal photons.

Summary on eX-scattering

The differential cross section of *eX*-scattering as a function of energy (E') and angle (θ) of scattered electron is:

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{4\alpha^2 E'^2}{q^4}\{R\}_{eX} \tag{30}$$

where

$$\{R\}_{e\mu\to e\mu} = \delta\left(\nu + \frac{q^2}{2m}\right)\cdot\cos^2\frac{\theta}{2} - \frac{q^2}{2m^2}\delta\left(\nu + \frac{q^2}{2m}\right)\cdot\sin^2\frac{\theta}{2}$$

$$\{R\}_{eP\to eP} = \delta\left(\nu + \frac{q^2}{2m}\right)\frac{G_E^2 + \tau G_M^2}{1+\tau}\cdot\cos^2\frac{\theta}{2} + 2\tau G_M\delta\left(\nu + \frac{q^2}{2m}\right)\cdot\sin^2\frac{\theta}{2} \tag{31}$$

$$\{R\}_{eP\to eX} = W_2\left(\nu, q^2\right)\cdot\cos^2\frac{\theta}{2} + 2W_1\left(\nu, q^2\right)\cdot\sin^2\frac{\theta}{2}$$

Bjorken's scalling

For scattering of point-like particles is valid:

$$mW_1^{point} = \frac{1}{2}\frac{Q^2}{2m\nu}\delta\left(1 - \frac{Q^2}{2m\nu}\right) = F_1(\omega)$$

$$\nu W_2^{point} = \delta\left(1 - \frac{Q^2}{2m\nu}\right) = F_2(\omega) \tag{32}$$

where $\omega = \frac{2m\nu}{Q^2}$ a $Q^2 = -q^2$.

The structure function for scattering of electron on point-like particle are only the functions of ratio ω.

The Bjorken scaling is demonstrated in Fig.3, where are presented the data of different collaboration (SLAC-MIT, BCDMS, H1 and ZEUS) for the structure function of proton, F_2. From Fig. 3 is seen that the shape of $\boldsymbol{F_2}$ in a good approximation is practically not changed neither if $\boldsymbol{Q^2}$ is changed in frame of 3 orders.

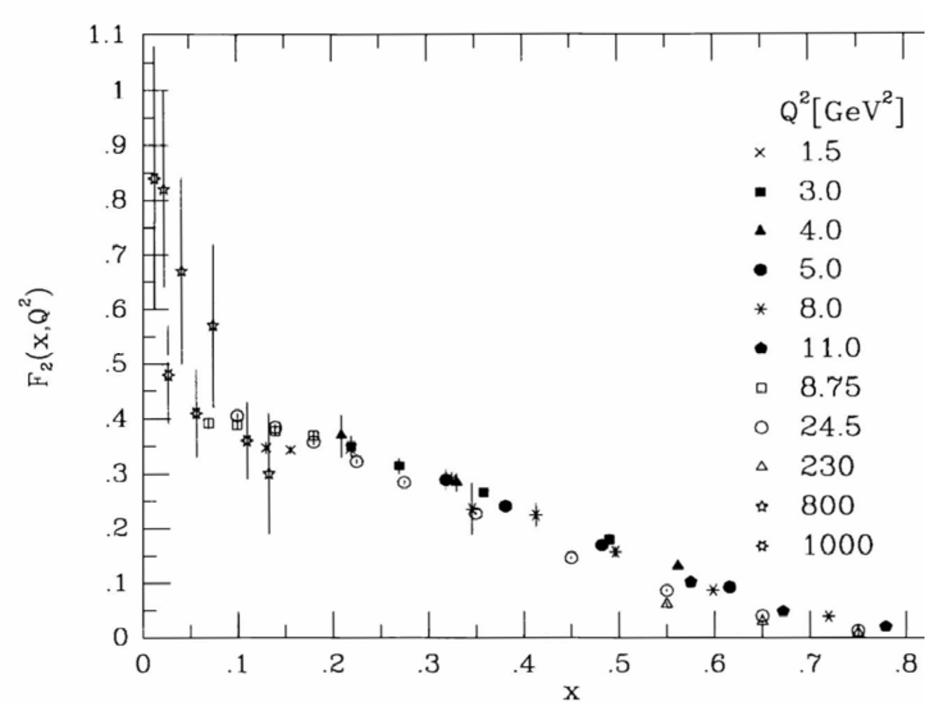


Fig. 3: The structure function F_2 as a function of x for different Q^2.

The Bjorken's scaling means that **virtual photon scatters on point like constituents of proton**. Else the structure function would depend on the ratio $\boldsymbol{Q/Q_0}$, where $\boldsymbol{1/Q_0}$ would present characteristic dimension of constituents.

Parton model

Let us consider ***ep***-scattering and assume that proton consist of point-like partons (Fig. 4):

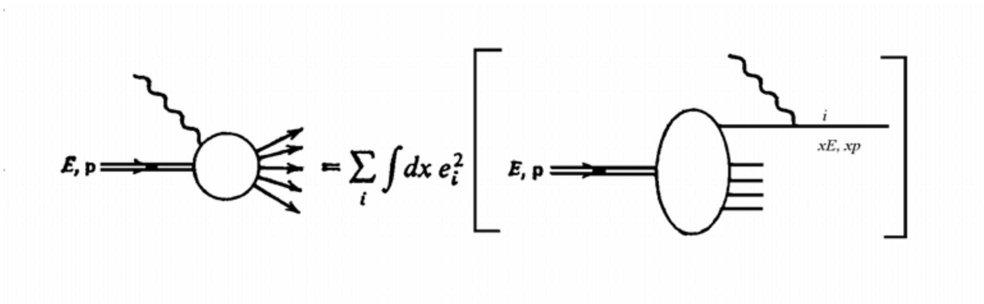
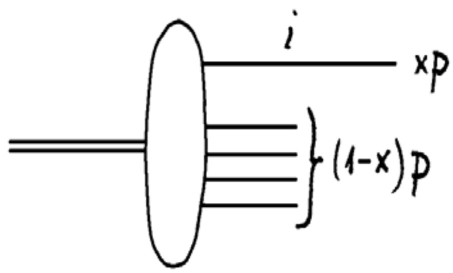


Fig. 4: The ep-scattering expressed through interaction of virtual photon with individual partons of proton.

The interaction of electron with proton is expressed through that of virtual photon with individual partons of proton which have the charge e_i and relative momentum x. Distribution of partons in momentum is given by the structure functions:

$$f_i(x) = \frac{dP_i}{dx} =$$



Structure function gives probability of a parton „***i***“ (that interacts with photon) to have the relative momentum x.

Normalization of structure functions:

$$\sum_i \int dx\, f_i(x) = 1 \qquad (33)$$

Kinematics of scattering on proton vs that on parton			
	Energy	momentum	mass
proton	E	P_L	M
parton	xE	xP_L	xM

If we consider interaction of electron with a parton with relative momentum x then in cross section for scattering will be present the structure functions of partons exhibit scaling (parton is point-like particle):

$$F_1^{part}(\omega) = \frac{Q^2}{4M\nu x^2}\delta\left(1 - \frac{Q^2}{2xM\nu}\right) = \frac{1}{2\omega x}\delta\left(x - \frac{1}{\omega}\right)$$
$$F_2^{part}(\omega) = \delta\left(1 - \frac{Q^2}{2xM\nu}\right) = \quad x\,\delta\left(x - \frac{1}{\omega}\right) \tag{34}$$

Where $\omega = \frac{2M\nu}{Q^2}$ characterizes the virtual photon.

Due to fact that the proton structure functions are proportional to the cross section of absorption of virtual photon, we can get them in such a way that we will sum the contributions of all partons:

$$F_2(\omega) = \sum_i \int dx\, e_i^2 f_i(x)\, x\,\delta\left(x - \frac{1}{\omega}\right)$$
$$F_1(\omega) = \frac{\omega}{2} F_2(\omega) \tag{35}$$

The presence of $\delta(x - 1/\omega)$ leads to the validity $x = 1/\omega$ and for the inelastic formfactors of proton we get:

$$\nu W_2(\nu, Q^2) \to F_2(x) = \sum_i e_i^2 x\, f_i(x)$$
$$MW_1(\nu, Q^2) \to F_1(x) = \frac{1}{2x} F_2(x) \tag{36}$$

It should be noted that the relative momentum x characterizes parton, while $1/\omega$ characterizes virtual photon. The equality $x = 1/\omega$ means that virtual photon will be absorbed by parton only at „the right“ value $1/\omega$ (= x).

It is important to realize that from physical point of view up to now the picture of interaction was the following: parton absorbed (virtual) photon and owing to that it changed its motion of state. At the same time the act of absorption of photon was an act of electromagnetic interaction. The strong interaction appeared in the process only in the „second“ phase as a consequence of colored charge of the hit parton.

Remark. The validity of the Callan–Gross relation, $F_2(x) = 2xF_1(x)$, is a consequence of the ½ spin of quarks. Assuming the parton spin ½ we derived the relation (29) for absorption of photon by parton. In the Bjöken limit ($\nu, Q^2 \to \infty$) the cross section of such absorption reads:

$$\sigma_L = \sigma_0\left(\left(1+\frac{\nu^2}{Q^2}\right)\frac{F_2(x)}{\nu} - \frac{F_1(x)}{M}\right) \approx \frac{\sigma_0}{M}\left(\frac{F_2(x)}{2x} - F_1(x)\right) = 0 \tag{37}$$

where $x = Q^2/(2M\nu)$. Hence parton of spin ½ cannot absorb longitudinal photon (only the transverse one). In case of scalar quark it would be just opposite.

Quarks in proton and neutron

The structure function of proton expressed through parton distribution functions (p.d.f.) (we suppose that in the structure of proton present *u*, *d*, *s*- quarks and corresponding anti-quarks):

$$\frac{1}{x}F_2^{ep}(x) = \left(\frac{2}{3}\right)^2\left[u^p(x)+\bar{u}^p(x)\right]+\left(\frac{1}{3}\right)^2\left[d^p(x)+\bar{d}^p(x)\right]+\left(\frac{1}{3}\right)^2\left[s^p(x)+\bar{s}^p(x)\right] \tag{38}$$

where $u^p(x)$ and $\bar{u}^p(x)$ are the probability distributions of quarks and anti-quarks in proton.

The structure function of neutron expressed through parton distribution functions:

$$\frac{1}{x}F_2^{en}(x) = \left(\frac{2}{3}\right)^2\left[u^n(x)+\bar{u}^n(x)\right]+\left(\frac{1}{3}\right)^2\left[d^n(x)+\bar{d}^n(x)\right]+\left(\frac{1}{3}\right)^2\left[s^n(x)+\bar{s}^n(x)\right] \tag{39}$$

where $u^n(x)$ and $\bar{u}^n(x)$ are the p.d.f. of quarks and anti-quarks in neutron.

Proton and neutron are the members of the same isospin doublet therefore their quark distributions are mutually connected:

$$\begin{aligned} u^p(x) &= d^n(x) \equiv u(x) \\ d^p(x) &= u^n(x) \equiv d(x) \\ s^p(x) &= s^n(x) \equiv s(x) \end{aligned} \tag{40}$$

Additional limitations on the quark structure functions follows from the fact that the quantum numbers of proton must be the same as the quantum numbers of the quark combination ***uud***. The conception of proton is the following: proton consists of 3 valence quarks surrounded by a big number of quark – anti-quark pairs $u_s\bar{u}_s,\ d_s\bar{d}_s,\ s_s\bar{s}_s$ etc. (so-called quark sea). At the same time, we assume that quarks ***u***, ***d*** and ***s*** are present in the quark sea with the same weights and distributions while the weights of heavy quarks are equal to zero. It leads to the following:

$$\begin{aligned} &u_s(x)=\bar{u}_s(x)=d_s(x)=\bar{d}_s(x)=s_s(x)=\bar{s}_s(x)=S(x)\\ &u(x)=u_v(x)+u_s(x)\\ &d(x)=d_v(x)+d_s(x) \end{aligned} \tag{41}$$

Taking into account that proton charge is 1, baryon number is 1 and strangeness is 0 we get:

$$\begin{aligned} &\int_0^1\left[u(x)-\bar{u}(x)\right]dx=2\\ &\int_0^1\left[d(x)-\bar{d}(x)\right]dx=1\\ &\int_0^1\left[s(x)-\bar{s}(x)\right]dx=0 \end{aligned} \tag{42}$$

Combining the obtained above we get:

$$\begin{aligned} &\frac{1}{x}F_2^{ep}=\frac{1}{9}\left[4u_v+d_v\right]+\frac{4}{3}S\\ &\frac{1}{x}F_2^{en}=\frac{1}{9}\left[u_v+4d_v\right]+\frac{4}{3}S \end{aligned} \tag{43}$$

Let us suppose that we “scan” proton with a high frequency photon $(\nu\to\infty)$. In case of low energy components ($x\approx 0$) of nucleon there will dominate the low energy $q\bar{q}$ -pairs from the „sea“ ***S(x)***. Therefore one can expect:

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)}\xrightarrow[x\to 0]{}1 \tag{44}$$

In the case of high energetic components there will dominate valence quarks, hence:

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)}\xrightarrow[x\to 1]{}\frac{u_v+4d_v}{4u_v+d_v} \tag{45}$$

At large ***x*** there are experimental reasons to assume that $u_v \gg d_v$ and the above mentioned ratio goes to ¼. As can be seen from Fig.5a both tendencies (44, 45) are experimentally observed.

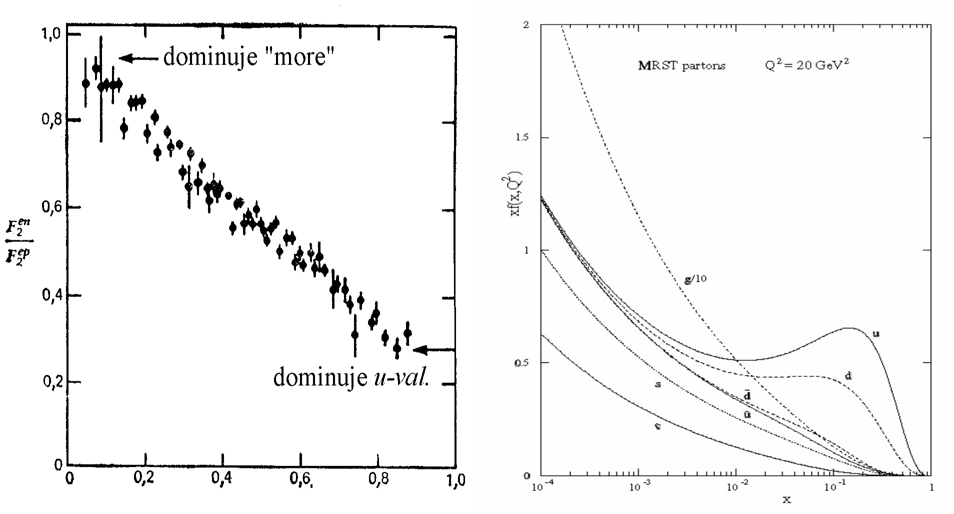


Fig. 5: (a) the ratio F_2^{en}/F_2^{ep} measured in deeply inelastic ***en***- a ***ep***- scattering as a function of ***x*** (data SLAC), (b) quarks and gluons in protons.

In Fig.5b is shown the experimentally status of proton structure obtained from the deeply-inelastic scattering data in experiments with fixed target at $Q^2 = 20\ GeV^2$. The results of experiments can be concluded as follows:

- Presence of sea s-quarks is suppressed compared to sea u- and d-quarks, this can be seen from comparison of distributions of ***s***-quark on one side and $\bar{u}$ and $\bar{d}$ on the second side. It means that the flavor SU(3)-symmetry is violated.
- In the region $x>0.01$ is observed asymmetry between $\bar{u}$ and $\bar{d}$ -quarks.
- Sea *c*-quarks are suppressed but present.
- Quarks carry about 50% of proton momentum – the rest is carried by gluons.