



Top Physics Studies at ATLAS/LHC

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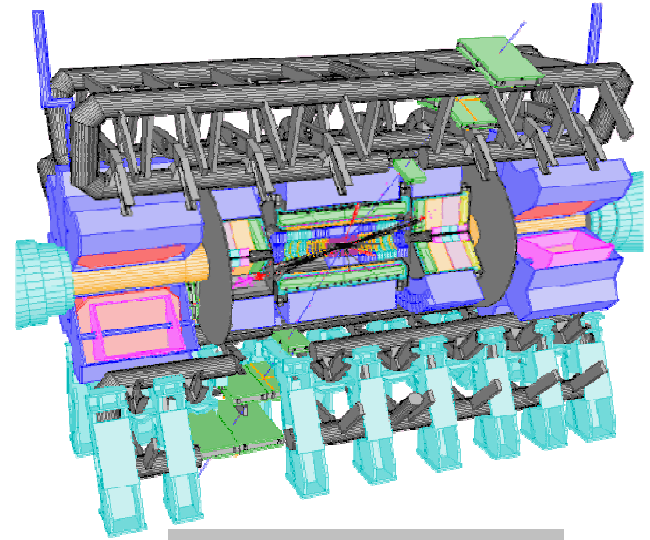
7 May 2004

S. Tokar, RTN Workshop 2004, Pisa



Topics in This Talk

- **Top production cross section**
precisious QCD tests
- **Top quark mass**
constraint on Higgs mass via EW physics
- **Single top production**
top partial width, V_{tb} , spin effects...
- **Top spin effects**
 - Top-antitop spin correlations
- **Anomalous couplings**
 - FCNC in top physics
- **Rare decays of top and exotics**
 - Search for charged Higgs, superpartners



ATLAS detector

LHC: pp 14 TeV

ATLAS per year

pp @ $L=10 \text{ fb}^{-1} \Rightarrow$

$10^7 t\bar{t}$ pairs

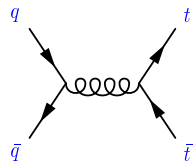
Start: 2007



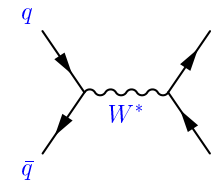
Top Quark Production at LHC

Strong $t \bar{t}$ pair production

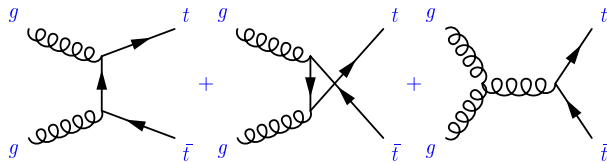
EW single top quark production



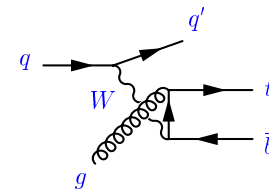
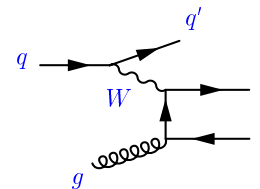
$q\bar{q} \rightarrow t\bar{t}$
(13%)



$q\bar{q} \rightarrow t\bar{b}$
(Drell-Yan)



$gg \rightarrow t\bar{t}$
(87%)



$qg \rightarrow q't\bar{b}$
(W-g fusion)

X-section (NLO+NLL) ≈ 830 pb

X-section (NLO) ≈ 300 pb

at CDF X-section ≈ 6 pb

Bonciani et al, NuclPhysB529(1998)424

CERN 2000-004 (eds. Altarelli, Mangano)

at LHC $\sqrt{s} = 14$ TeV



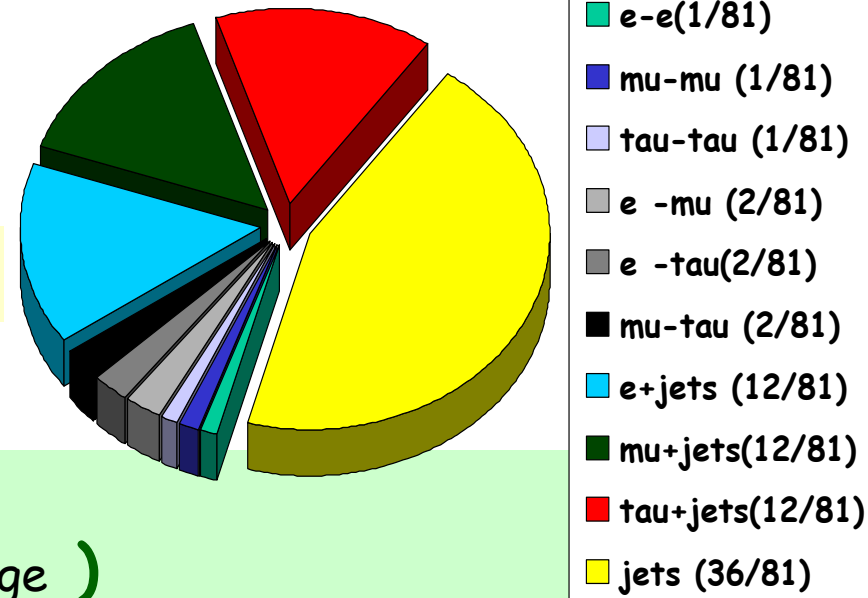
Top Quark Decay

SM: by far dominant $t \rightarrow bW$

$$\frac{\Gamma(t \rightarrow bW)}{|V_{tb}|^2} \approx 0.807 \times \frac{G_F m_t^3}{8\pi\sqrt{2}} = 1.42 \text{ GeV}$$

$$\tau_{\text{top}} \approx 5 \times 10^{-25} \text{ sec} \ll \tau_{\text{hadr}} (10^{-23} \text{ sec})$$

$$(m_t = 175 \text{ GeV}, M_W = 80.4 \text{ GeV}, \alpha_s = 0.03475)$$



Top decays before hadronization !!!

- No $t\bar{t}$ -bar bound states (gluon exchange)
- t, W helicity from SM V-A (no depolarization via hadronization)

$t\bar{t}$ -bar samples
defined via W decays

- Dilepton channels ($ee, e\mu, \mu\mu$)
topological variables and b-tagging
- Lepton + jets ch. ($e+jets, \mu+jets$)
topological analysis and b-tagging
- All hadronic channel



$t\bar{t}$ Production Cross Section

Big $m_t \Rightarrow \alpha_s(m_t) \sim 0.1 \Rightarrow$ pExpansion converges rapidly :

- $t\bar{t}$ cross section is a test of QCD predictions
(Inclusive and differential cross sections)
- Accurate Xsection $\Rightarrow$ indirect determination of m_t
(in SM is expected: $\Delta m/m \approx \Delta \sigma(t\bar{t})/\sigma(t\bar{t})$)
- Rapidity distr. asymmetries of t and $\bar{t}$ are sensitive to light-quark PDF
- A discrepancy may indicate a new physics
 - Production via a high mass intermediate state
 - Non Wb decay model

ATLAS: Statistical uncertainties < 1%

$\rightarrow$ Systematics (Exp.& Theo.) will be dominant



$t \bar{t}$ Production Cross Section

Theory for top X-section (Inclusive and differential):
 NNLO-NNLL (Kidonakis et al., PRD68,114014 (2003))

- Factorization

$$\sigma = \sum_{i,j} \int dx_1 dx_2 F_i^{(1)}(x_1, \mu_F) F_j^{(2)}(x_2, \mu_F) \hat{\sigma}_{ij}(s; \mu_F, \mu_R)$$

Usual choice: $\mu_F = \mu_R = \mu \in (m_t/2, 2m_t)$

- Total Partonic Xsection:

$$\hat{\sigma}_{ij} = \frac{\alpha_s^2(\mu)}{m^2} \sum_{n=0}^{\infty} \left(4\pi\alpha_s(\mu) \right)^n \sum_{k=0}^n f_{ij}^{(n,k)}(\eta) \ln^k \left(\frac{\mu^2}{m^2} \right)$$

$$\eta = \frac{s}{4m^2} - 1$$

$$s = (p_i + p_j)^2$$

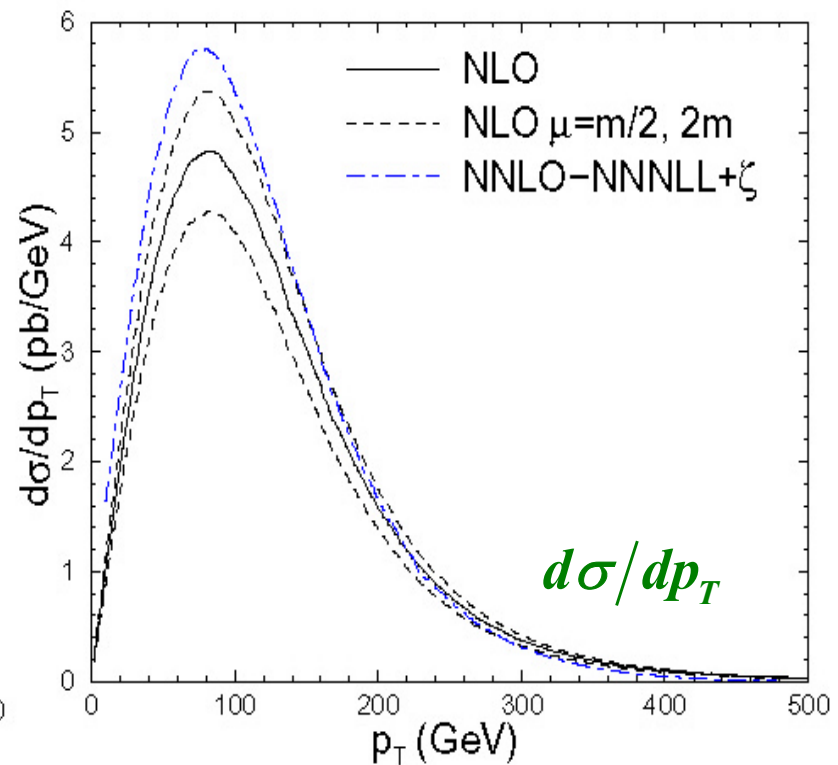
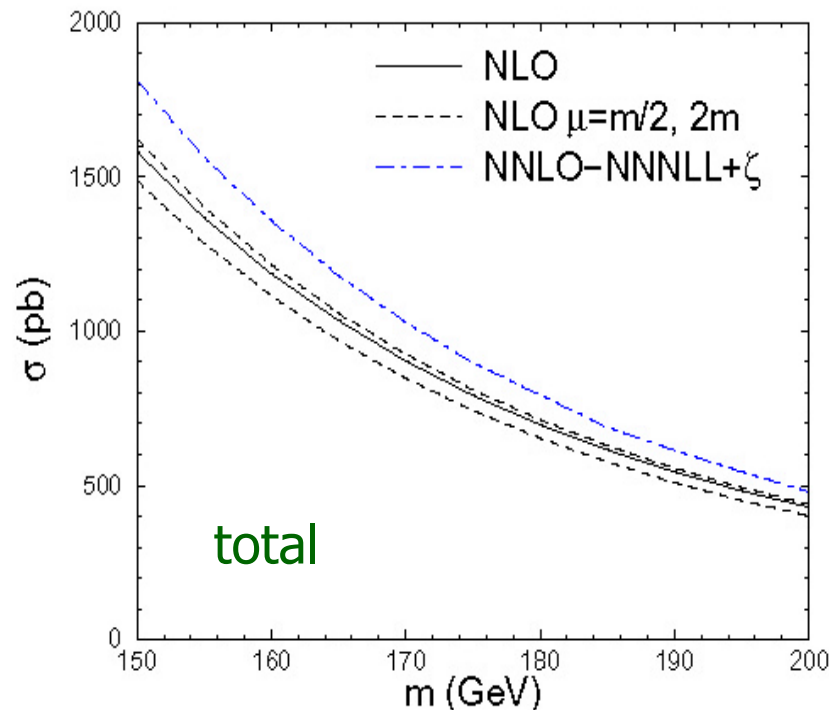
Progress at MC: radiative gluon corrections included: **MCatNLO**
 (Frixione et al, hep-ph/0311223)



$t\bar{t}$ Cross Section at 14 TeV

$pp \rightarrow t\bar{t}$ $S^{1/2}=14$ TeV $m=175$ GeV

$pp \rightarrow t\bar{t}$ $S^{1/2}=14$ TeV



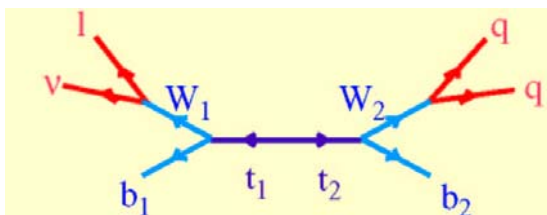
NNLO: uncertainty from scale ($m_+/2, 2m_+$) < 3% !!!

(N.Kidonakis, hep-ph/0401147)



Top Mass measurement at ATLAS/LHC

Lepton +jets channel



**Borjanovic et al.,
SN-ATLAS-2004-040**

At prod. level: $S/B=10^{-5}$

⇒ cuts applied:

$p_T(l) \geq 20 \text{ GeV}, E_T^{\text{miss}} \geq 20 \text{ GeV}$

$\geq 4 \text{ jets } (p_T > 40 \text{ GeV}, |\eta| \leq 2.5, \Delta R = 0.4)$

⇒ $S/B \sim 78$, 8700 tt events /10fb⁻¹, 2b-tag

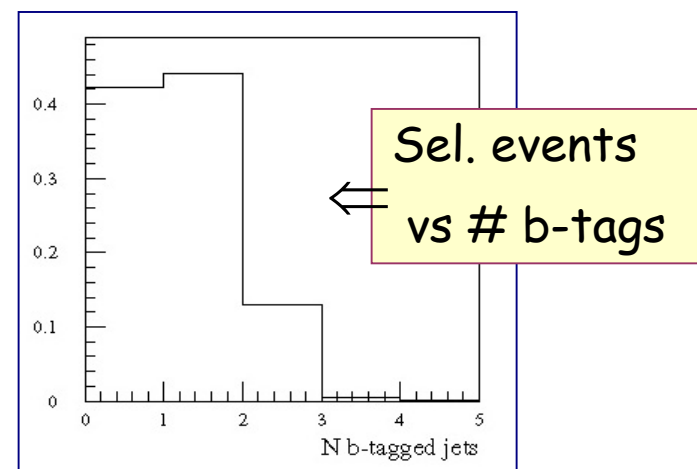
Purity 55% for 1 b-tagged, eff=2.5 %

66% for 2 b-tagged, eff=1.2 %

Top mass window : $\pm 35 \text{ GeV}$

Process	Cross section (pb)	Efficiency (%)
tt signal	250	3.5
$bb \rightarrow lv + jets$	2.2×10^6	3×10^{-8}
$W + jets \rightarrow lv + jets$	7.8×10^3	2×10^{-4}
$Z + jets \rightarrow l l^+ + jets$	1.2×10^3	6×10^{-5}
$WW \rightarrow lv + jets$	17.1	7×10^{-3}
$WZ \rightarrow lv + jets$	3.4	1×10^{-3}
$ZZ \rightarrow l l^+ + jets$	9.2	3×10^{-3}

Signal vs Bkgd: x-section and sel. efficiency



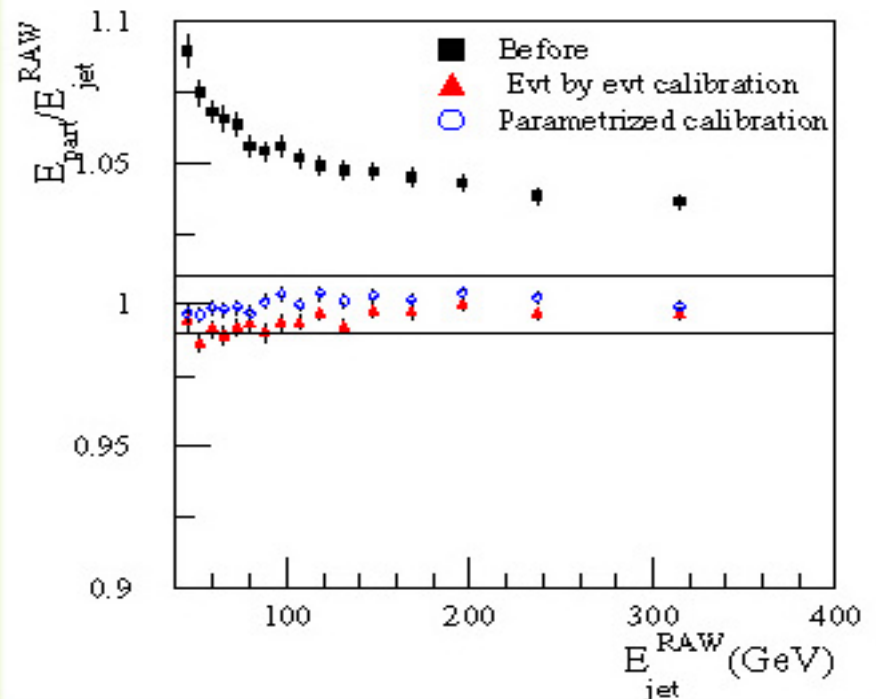
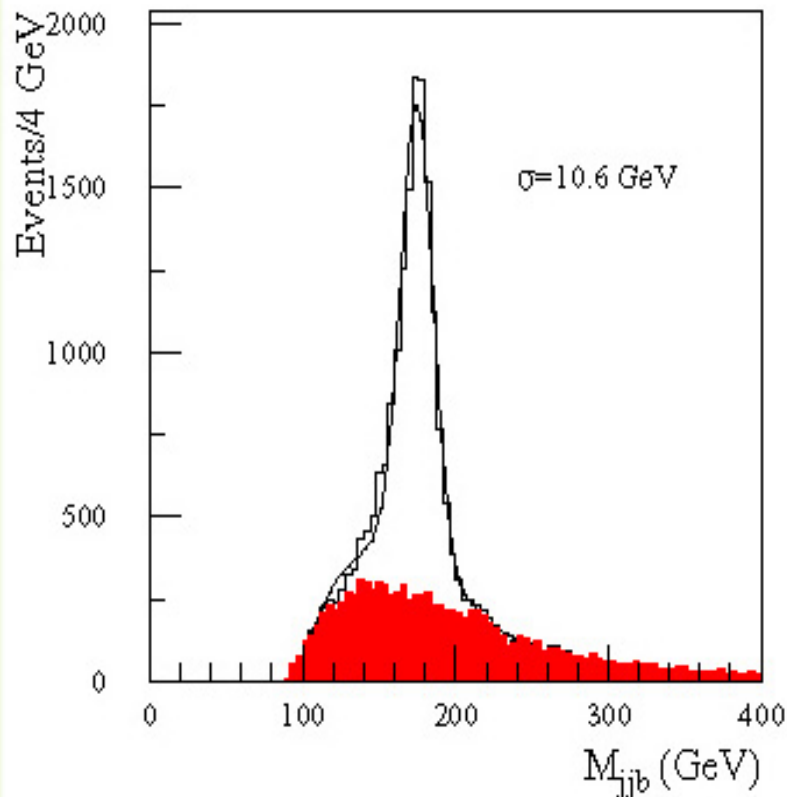
Top mass in lepton + jets channel



Jet energy correction

$$K = 1.029 + 3.20/E_{\text{raw}}$$

Needed to minimize Δm_t



⇐ Invariant mass of jjb
(b-jets calibrated using Z+b events,
 M_W window: ± 20 GeV)

For 10 fb⁻¹ sample: Δm_t

- statistical error 100 MeV
- systematics ~ 1.3 GeV (FSR,...)



Top mass in lepton + jets channel

Leptonic part - mass reconstructed via :

- missing transverse energy ($E_T^\nu = E_T^{miss}$)
- constraint $m(l\nu) = M_W$ for neutrino p_z

Result: two leptonic top masses due to twofold ambiguity in neutrino p_z

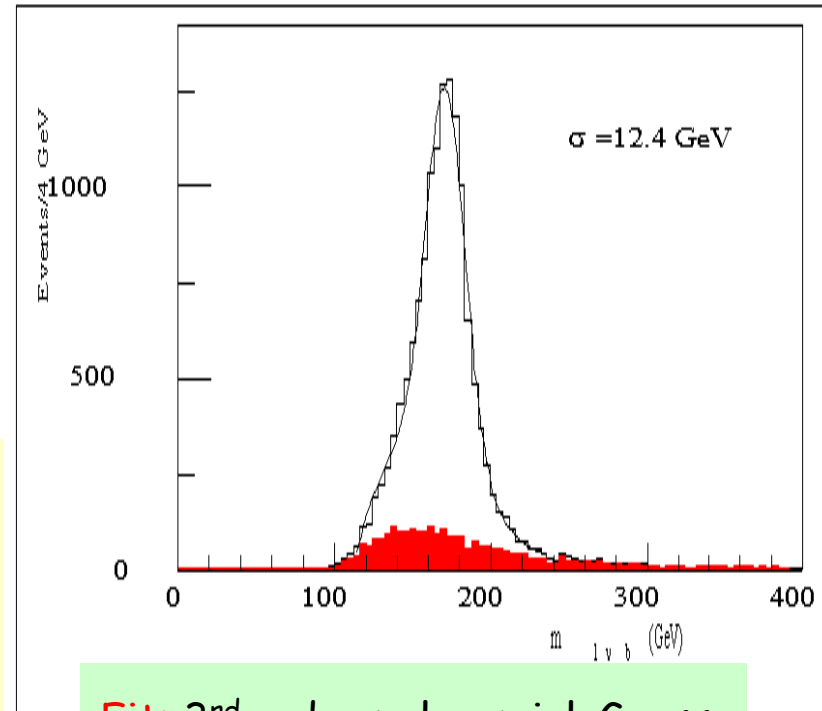
Kinematic fit (event by event):

Lepton and jets energy, jets direction (ϕ, η), neutrino p_x, p_y, p_z vary freely (in errors inter.)

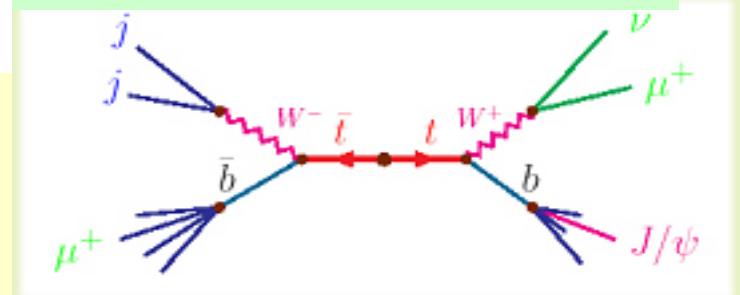
χ^2 uses constraints $M_{jjb} = m_t$, $M_{l\nu b} = m_t$, $M_{jj} = M_W$
 $\Rightarrow m_t$ find by fit, $\chi^2 < 4 \rightarrow$ purity $> 80\%$

Promising: *$l + J/\psi$ channel*

- Strong correlation between m_t & $m(l, J/\psi)$
- $BR = 3.2 \times 10^{-5}$ (2700 ev/100fb $^{-1}$, sel. $\epsilon \approx 16\%$)
- non-sensitive to jet energy, $S/B \approx 55$



Fit: 3rd order polynomial+Gauss



$t \rightarrow W^+ b$, $W^+ \rightarrow l \nu$, $b \rightarrow J/\psi X$



Top mass in di-lepton channel

Selection: 2 isolated leptons ($p_T > 35, 25 \text{ GeV}$)

High missing $E_T (> 40 \text{ GeV})$

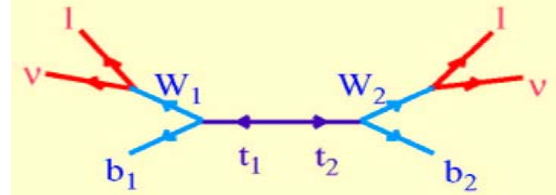
2 b-jets with $p_T > 25 \text{ GeV}$

Neutrino momenta found from:

$$\sum p_x = \sum p_y = 0, \quad m_{l^+ \nu} = M_W, \quad m_{l^- \tilde{\nu}} = M_W,$$

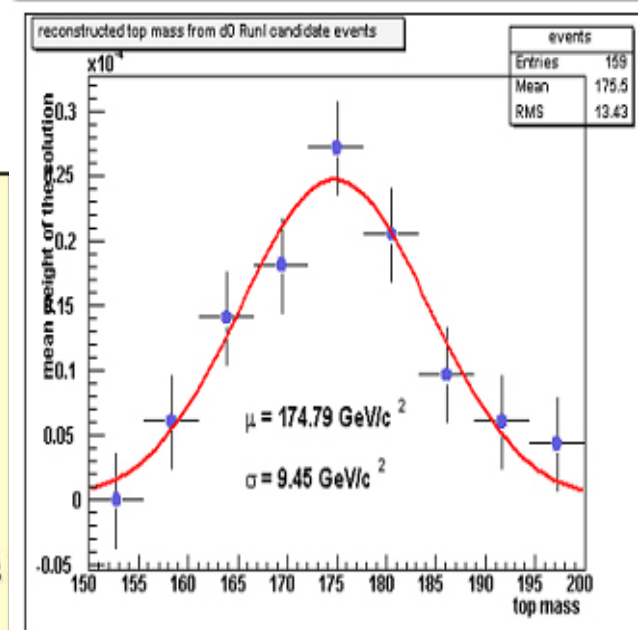
$$m(l^+ \nu b) = m_t, \quad m(l^- \tilde{\nu} b) = m_t \quad m_t \text{ is known}$$

$$\text{For weight} \Rightarrow m_t^2 = M_W^2 + 2 \langle m_{lb}^2 \rangle / (1 - \langle \cos \theta_{lb} \rangle)$$



For each event: full reconstruction is done for different input top masses and mean weight is calculated for each input m_t

Source of uncertainty	$\delta M_{Top} \text{ (GeV)}$	Comments
Statistic & reconstruction method	0.3	
b-jet energy scale	0.6	1% error
b-quark frag.	0.7	$(\epsilon_b = -0.006) - (\epsilon_b = -0.035)$
ISR	0.1	20%(ON-OFF)
FSR	0.6	20%(ON-OFF)
Parton distribution function	1.2	Mass shift from 2 different sets
Total	1.7	

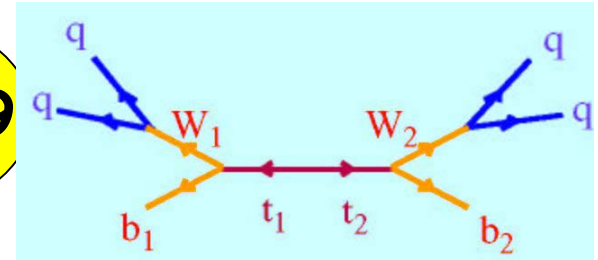




Top mass in all jet channel

Selection:

≥ 6 jets with $p_T > 40$ GeV, ≥ 2 b-jets $\Rightarrow$ **S/B = 1/19**
Small missing E_T



Kinematic fit (two steps)

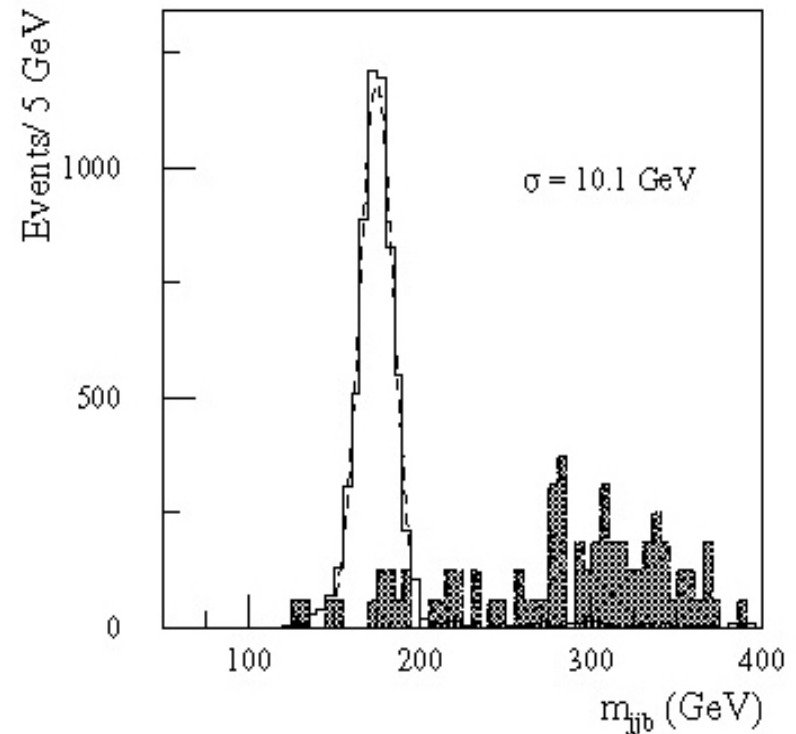
- **W's selection and reconstruction with W mass constraints**
- **Both W candidates are combined with the b-jets.**

Combination chosen using top mass constraint: $m(t_1) = m(t_2)$

Top mass window (130-200 GeV)
 $\Rightarrow$ **S/B = 6/1**

High PT subsample $\Rightarrow$ Selection

- **$p_T(t) > 200$ GeV**
- **3300 evts/ $10 \text{ fb}^{-1} \Rightarrow$ S/B = 18/1**
- **$\Delta m^{\text{stat}}(t) = 0.2 \text{ GeV}/c^2$**
- **$\Delta m^{\text{sys}}(t) = 3 \text{ GeV}/c^2$**





Top mass and EW precision physics

Atlas top mass systematics ($L=10\text{fb}^{-1}$): jet energy scale, b-jet frag., ISR, FSR, comb. Bckgd. All mass method combined $\Rightarrow$

$$\Delta m_t(\text{stat}) = \pm 0.1 \text{ GeV} \quad \Delta m_t(\text{syst}) = \begin{cases} +0.6 \\ -0.8 \end{cases} \text{ GeV}$$

ISR, FSR via Pythia $\Rightarrow$ Realistic: $\Delta m_t \approx 1 \text{ GeV}$

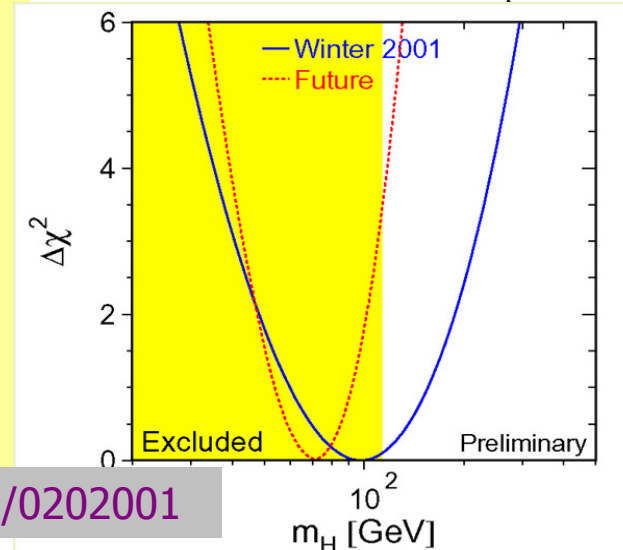
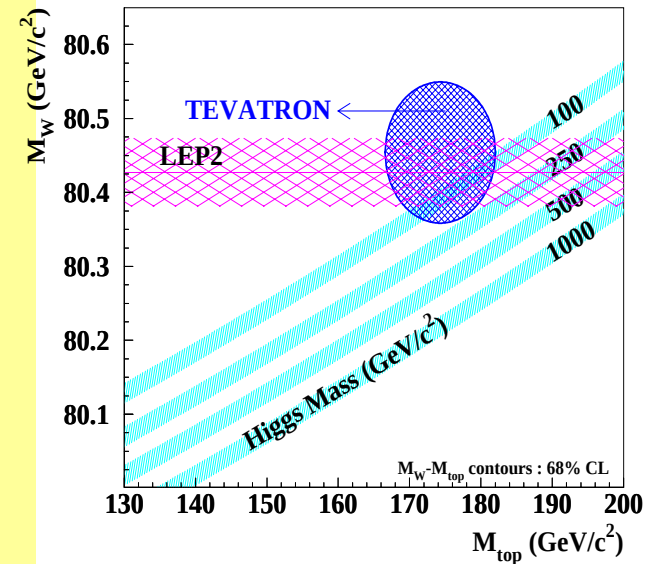
Masses of top, W and Higgs are bounded by

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi\alpha}{\sqrt{2}G_F} (1 + \Delta r), \quad \Delta r = \Delta\alpha + \frac{s_W}{c_W} \Delta\rho + (\Delta r)_{nl}$$

$\Delta r \equiv$ rad. corrections ($\Delta\rho \sim m_t^2$, $(\Delta r)_{nl} \propto \ln M_H$)

Precise M_W and $m_t \Rightarrow$ constraint on M_H !

present $\rightarrow$ LHC: Δm_t : $5 \rightarrow 1 \text{ GeV}$, ΔM_W : $33 \rightarrow 15 \text{ MeV}$



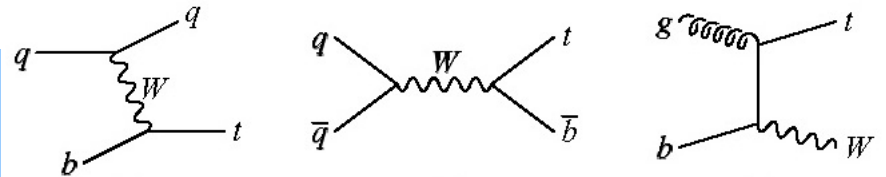
Grunwald et al, hep-ph/0202001



Single top production

Production via weak forces

- Xsection $\sim |V_{tb}|^2$
(direct measurement of V_{tb})
- Significant bckgd to Higgs signal
- Single top -100% polarization
(test of V-A structure of EW)
- Possible new physics



t-channel

s-channel

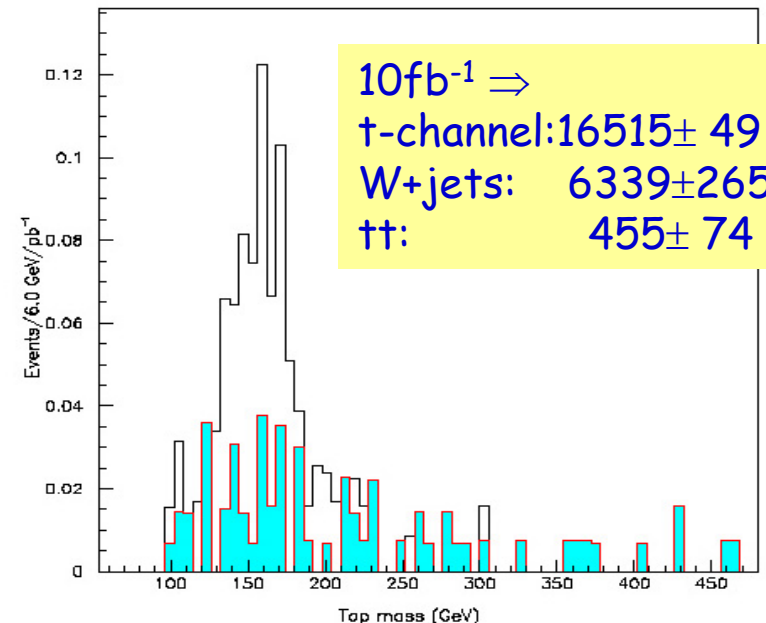
association
production

245 $\pm$ 27 pb 10.7 $\pm$ 0.7 pb
(at LHC 14 TeV, NLO)

51 $\pm$ 9 pb

Selection criteria

- Only 1 isolated lepton ($p_T > 20$ GeV, $|\eta| < 2.5$)
- miss- $p_T > 20$ GeV, $50 < m_T(l+\nu) < 100$ GeV
- exactly 2 jets: ($p_T > 20$ GeV, $|\eta| < 4$)
1 jet with $p_T > 20$ GeV, $|\eta| < 2.5$
1 jet with $50 < p_T < 100$ GeV, $2.5 < |\eta| < 4$
- Exactly one b-tagged jet (reduces $t\bar{t}$ -bkgd)
- Two jet invariant mass $\in (80, 100)$ GeV
(rejects WZ events)





$t\bar{t}$ spin correlations

Top decays before hadronization $\Rightarrow$

- Spin of top is not diluted
- Gluon emission unlikely to do $\Delta S \neq 0$

Imprint of $t\bar{t}$ production spin:

Angular distrib. of top decay prod's

Considered parton reactions:

$$gg, q\bar{q} \rightarrow t\bar{t} + X \rightarrow b\bar{b} + 4f + X \quad f \equiv q, l, \nu$$

$$|M^{(\lambda)}|^2 \propto \text{Tr}[\rho R^{(\lambda)} \bar{\rho}], \quad \lambda = gg, q\bar{q} \quad (\text{narrow w.})$$

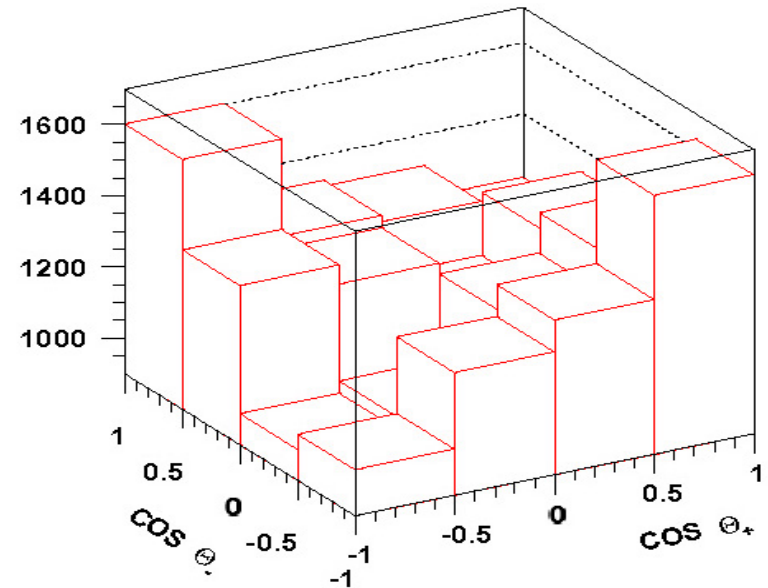
Decay of polarized t quark

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta_f} = \frac{1}{2} (1 + \kappa_f \xi_t \cdot \hat{q}_f)$$

$\theta_f \equiv$ angle (direction $\hat{q}_f$ of f , polarisat. ξ_t)

$\kappa_f \equiv$ Correlat. coef. for V-A current

for anti- t decay: $\kappa_f \rightarrow -\kappa_f$



Most promising:

Dilepton l^+l^- angular distribution

$$\frac{1}{\sigma} \frac{d^2\sigma}{d\cos\theta_+ d\cos\theta_-} = \frac{1 + C\kappa_{l^+}\kappa_{l^-} \cos\theta_+ \cos\theta_-}{4}$$

$\theta_+(\theta_-) \equiv$ angle $(n_t(n_{\bar{t}}), l^+(l^-)\text{direction})$

$t\bar{t}$ CMS $t(\bar{t})LS$

$$\kappa_{l^+}\kappa_{l^-} = -1, \quad C = 0.332 \text{ (SM)}$$



Top and CP violation

- CKM phase $\rightarrow$ only tiny effect on t-pair production and decay
- Non SM CP-violating interactions $\rightarrow$ in prod. density matrix (R) 2 types CP-odd spin-momentum correlations:

$$\hat{k}_t \cdot (s_t - s_{\bar{t}}), \quad \hat{k}_t \cdot (s_t \times s_{\bar{t}}), \quad \hat{k}_t \rightarrow \hat{p}$$

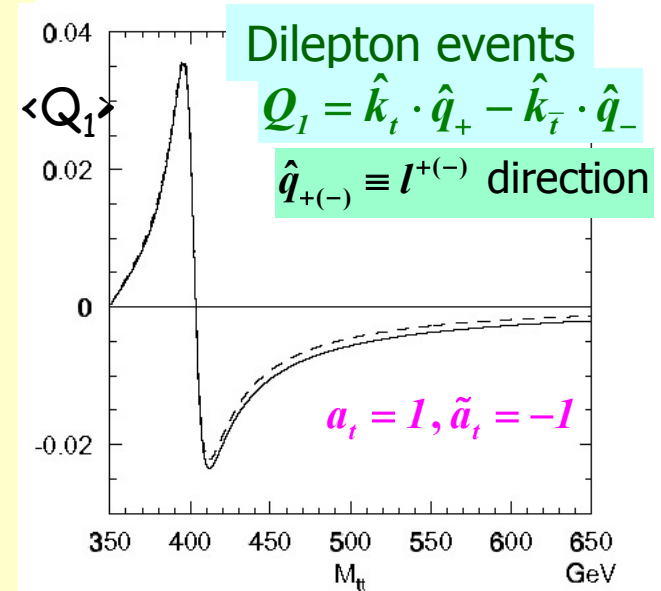
$\hat{k}_t \equiv$ top direction, $\hat{p} \equiv$ init. quark direction

Examples:

- In MSSM fermion-sfermion-neutralino interactions $\rightarrow$ CP violating phases from SUSY breaking terms
- Extended scalar sector $\rightarrow$ via non-degenerate neutral Higgs bosons with undefined CP parity.

Coupling of Higgs (φ) with top: $\sim m_t (a_t \bar{t} t + i \tilde{a}_t \bar{t} \gamma_5 t) \varphi$
(in SM $a_t = 1, \tilde{a}_t = 0$)

$gg \rightarrow \varphi \rightarrow t\bar{t}$ becomes resonant at $m_\varphi \sim 2 m_t$ or above



Bernreuther et al,
hep-ph/9812387

Can be also employed:

Asymmetry:

$$A(Q_1) = \frac{N_{ll}(Q_1 > 0) - N_{ll}(Q_1 < 0)}{N_{ll}}$$



Top quark anomalous interactions

- Top Xsection known with ~10% accuracy
- No top hadrons
- Top decay via pure V-A weak interaction
- Only one significant decay channel: $t \rightarrow Wb$



Top quark a unique place for a new physics behind SM

New physics from symmetry breaking at scale Λ (~1TeV)?

Anomalous $gt\bar{t}$ couplings: Cross section of $q\bar{q} \rightarrow t\bar{t}$ will have terms for

- anomalous chromomagnetic and chromoelectric dipole moments
- Retrieved from $|t\bar{t}|$ (top pair decay) observables:

Choi et al, PhysLettB415(1997)67 $\Rightarrow$

$$T_{33} = 2(p_{\bar{t}} - p_t)_3(p_{\bar{t}} \times p_t)_3, \quad A_E = E_{\bar{t}} - E_t$$

$$Q'_{33} = 2(p_{\bar{t}} + p_t)_3(p_{\bar{t}} - p_t)_3 - \frac{2}{3}(p_{\bar{t}}^2 - p_t^2)$$

Anomalous Wtb couplings

- Can be probed in top pair and single top production.
- 4 form factors describe Wtb - two are $\frac{1}{2}$ (from SM) and 2 to be analyzed:

$$F_{L(R)2} = \frac{2M_W}{\Lambda} \kappa_{tb}^W \left(-f_{tb}^{W*} - (+)ih_{tb}^{W*} \right)$$

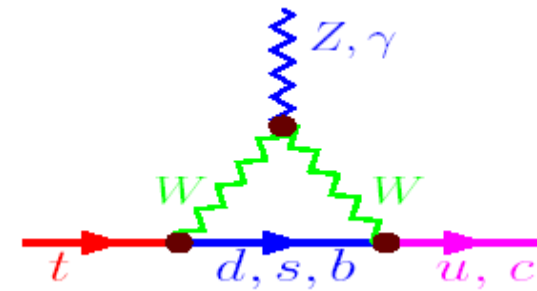
$\Rightarrow$ Boos et al,EFJC11(1999)473



FCNC in Top Quark Physics

FCNC couplings tVc and tVu ; $V = g, \gamma, Z$

- Absent at tree-level and highly suppressed in SM
- Present through loop contributions
- Observation of top quark FCNC processes
➔ **New Physics!**



CDF & LEP2
Present Limits

BR < 17%

BR < 3.2%

BR < 18%

$t \rightarrow$	SM	two-Higgs	SUSY
$g q$	5×10^{-11}	10^{-6}	10^{-3}
γq	5×10^{-13}	10^{-6}	10^{-5}
$Z q$	$\sim 10^{-13}$	10^{-9}	10^{-4}



FCNC in top quark physics

- Top decay widths and BR for anomalous couplings(=1) and for SM ($v=250 \text{ GeV}$, $m_t=175 \text{ GeV}$, $\Lambda=1 \text{ TeV}$, Han et al, NuclPhysB454(1995)527)

	Top decay mode				
	W^+b	$(c+u)g$	$(c+u)\gamma$	$(c+u)Z_\gamma$	$(c+u)Z_\sigma$
FCNC coupling		1	1	1	1
$\Gamma(\text{GeV})$	1.56	2.86	0.17	2.91	0.14
B	0.20	0.37	0.022	0.38	0.018
FCNC coupling		8×10^{-6}	3×10^{-6}	4×10^{-7}	
$\Gamma_{\text{SM}}(\text{GeV})$	1.56	8×10^{-11}	8×10^{-12}	2.2×10^{-13}	
B_{SM}	1	5×10^{-11}	5×10^{-13}	1.5×10^{-13}	

	<i>Tevatron</i>	<i>LHC</i>
$\sqrt{s} \text{ (TeV)}$	2.0	14.0
$L(\text{fb}^{-1})$	2	10
$\kappa_{tu}^g(2 \rightarrow 1)$	0.019	0.0033
$\kappa_{tu}^g(2 \rightarrow 2)$	0.026	0.0061
$\kappa_{tu}^g(2 \rightarrow 1)$	0.062	0.0084
$\kappa_{tu}^g(2 \rightarrow 2)$	0.092	0.013

Limits on anomalous couplings $\Rightarrow$

Indirect constraint (from $K_L \rightarrow \mu\mu$, $\Delta m(K_L - K_S, \dots)$): $\sqrt{|v_{tu}^Z|^2 + |a_{tu}^Z|^2} < 0.15$

CDF results (decays $t \rightarrow \gamma c(u)$ and $t \rightarrow Z c(u)$): $\sqrt{|v_{tq}^Z|^2 + |a_{tq}^Z|^2} < 0.73$, $\kappa_{tq}^\gamma < 0.78$



Rare SM top decays

- Direct measurement of V_{ts} , V_{td} via decays $t \rightarrow sW$, $t \rightarrow dW$

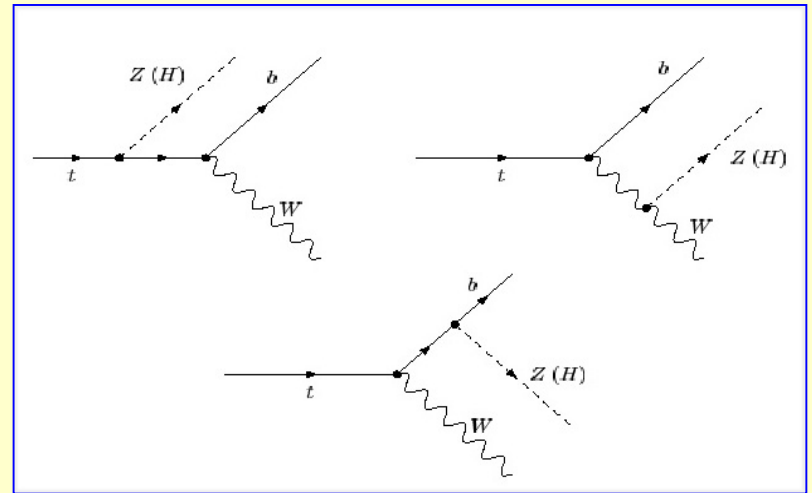
- Decay $t \rightarrow bWZ$ is near threshold

$$(m_t \sim M_W + M_Z + m_b) \Rightarrow$$

$$\text{BR}_{\text{cut}}(t \rightarrow bWZ) \approx 6 \times 10^{-7}$$

(cut on $m(ee)$ is $0.8 M_W$)

- Decay $t \rightarrow cWW$ suppressed by GIM factor m_b^2 / M_W^2 $\text{BR}(t \rightarrow cWW) \sim 1 \times 10^{-13}$



- If Higgs boson is light: $t \rightarrow bWH$
- FCNC decays: $t \rightarrow cg$, $t \rightarrow c\gamma$, $t \rightarrow cZ$ (BR: 5×10^{-11} , 5×10^{-13} , 1.3×10^{-13})
- Semi-exclusive t-decays $t \rightarrow bM$

(final state 1 hadron recoiling against a jet:

$$\text{BR}(t \rightarrow b\pi) \approx 4 \times 10^{-8}, \text{BR}(t \rightarrow bD_s) \approx 2 \times 10^{-7})$$



Non-SM Decays of Top

- 4th fermion family

Constraints on $|V_{tq}|$ relaxed: $BR(t \rightarrow W^+ b' (\rightarrow W^- c)) \sim 10^{-3}$ at $m_{b'} = 100 \text{ GeV}$

- Supersymmetry (MSSM)

- Observed bosons and fermions would have superpartners $\Rightarrow$
2-body decays into squarks and gauginos ($t \rightarrow H^+ b$)

$$(t \rightarrow \tilde{t}_1 \tilde{g}, t \rightarrow \tilde{b}_1 \tilde{\chi}_1^+, t \rightarrow \tilde{t}_1 \tilde{\chi}_1^{+0})$$

- Big impact on 1 loop FCNC

- two Higgs doublets

- $H^\pm$ LEP limit 77.4 GeV (LEP WG 2000)
- Decay $t \rightarrow H^+ b$ can compete with $t \rightarrow W^+ b$
- 5 states (h^0, H^0, A^0, H^+, H^-) survive after giving W & Z masses
- $H^\pm$ couples to heaviest fermions $\Rightarrow$ detection through breakdown of $e / \mu / \tau$ universality in $t \bar{t}$ production



Top quark charge

CDF, D0+precision EW data do not exclude : "top" quark is an exotic quark with charge $-4/3$ (Chang et al. PRD59, 091503)

Exotics: $t \rightarrow W^- b$ (in SM: $t \rightarrow W^+ b$)

Top charge determination:

- by measuring charges of top decay products

$$t \rightarrow W^+ b \text{ (W charge: } W^+ \rightarrow l^+ \nu, \quad Q_{bjet} = \sum_i Q_i |\vec{j} \cdot \vec{p}_i| / \sum_i |\vec{j} \cdot \vec{p}_i|)$$

- Via radiative $t \bar{t}$ events (sensitive to Q_t)

Analysis for ATLAS (10fb^{-1}) (Ciljak et al, Atl-Phys-2003-35):

- $l^+(l^-)$ -bjet association criteria can be found to distinguish between mean $Q(b_{jet}l^+)$ and $Q(b\text{-jet}(l^-))$.
- Radiative top production ($pp \rightarrow t\bar{t}\gamma$) can be used to measure foton-top coupling ($\sigma \sim Q_t^2$) $\Rightarrow$

Integrated Xsection: $\sigma^{\text{seen}}(Q=2/3) = 7.8\text{fb}^{-1}$, $\sigma^{\text{seen}}(Q=2/3) = 24.8\text{fb}^{-1}$

Background: $\sigma^{\text{seen}} = 6.5\text{fb}^{-1}$

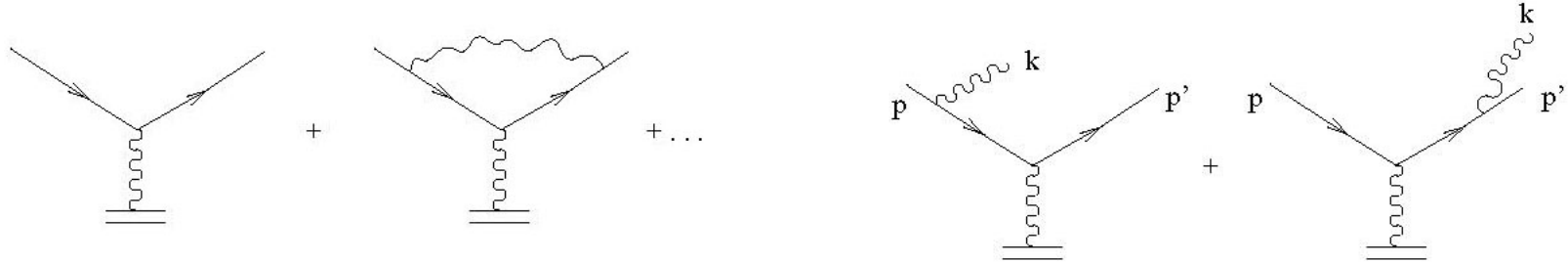


Conclusions

- Top quark physics is rich even at low luminosity LHC ($10\text{fb}^{-1}/\text{year}$)
 - SM: EW and QCD tests
 - Behind SM: probe SUSY
 - Determination of top characteristics (X-section, mass, width, charge..)
 - Important background
 - Top quark production (pair and single) is main background to processes with multi lepton + jets in final state e.g. Higgs physics, SuSy
 - High statistics studies at LHC ($100\text{fb}^{-1}/\text{year}$)
 - $\Delta m_t \leq 1 \text{ GeV}$
 - BR for FCNC $tVq \approx 10^{-3} - 10^{-6}$
 - Measurement of CKM V_{tq}
- ❖ We are looking forward to 2007...(LHC starts)



NLO, NNLO+LL, NLL, ...



Jet not seen if $m_{qg}^2 < M^2 = ys$

$$\sigma_{2j} = \sigma_0 \left[1 - \frac{\alpha_s C_F}{\pi} \ln^2 y + \frac{1}{2!} \left(\frac{\alpha_s C_F}{\pi} \ln^2 y \right)^2 + \dots \right] = \sigma_0 \exp \left(- \frac{\alpha_s C_F}{\pi} \ln^2 y \right)$$

$$\sigma_{3j} = \sigma_0 \frac{\alpha_s C_F}{\pi} \ln^2 y \times \exp \left(- \frac{\alpha_s C_F}{\pi} \ln^2 y \right)$$